Automata Theory and Dynamic Programming

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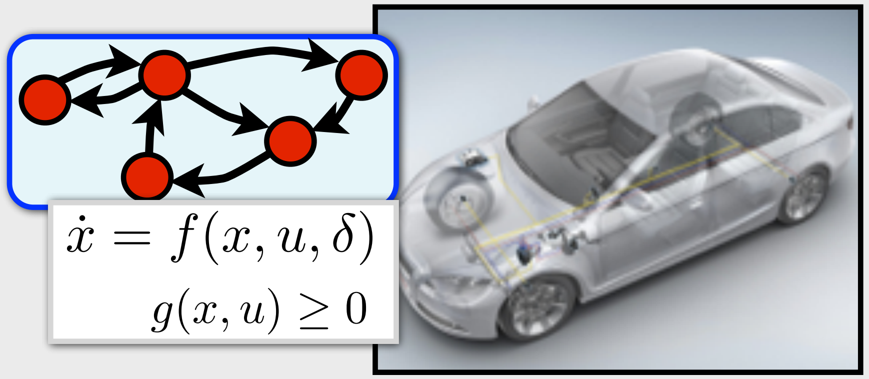
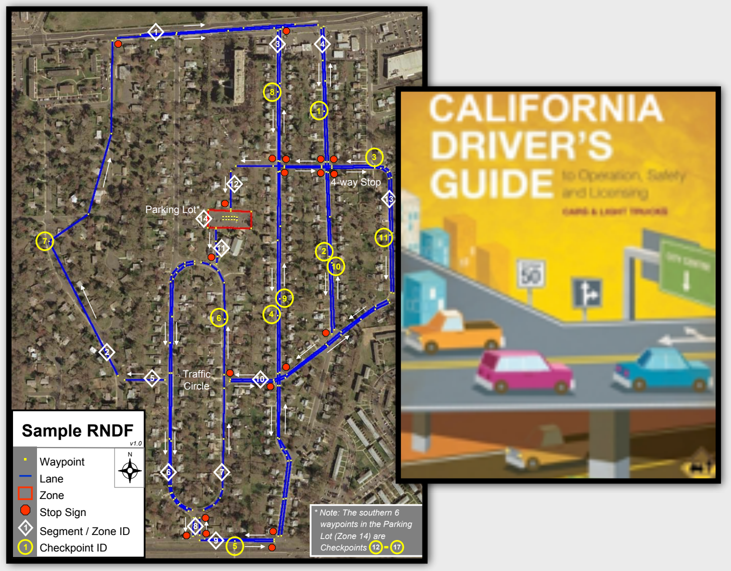
A Synthesis Problem

Given:

- ***System model***
  - both continuous & discrete evolution
  - actuation limitations
  - modeling uncertainties & disturbances



- ***Specifications***
  - high-level requirements
  - optimality criteria



Automatically synthesize a control protocol that

- manages the system behavior and
- is **provably correct** with respect to the specifications and **optimal**.

Outline

1. Abstraction-based synthesis

2. Approximate Dynamic Programming

3. Learning from expert demonstrations

Detour: Specifying Behavior with Temporal Logic

(only a dialect in a large family of languages)

Propositional Logic	$\wedge$ (and) $\vee$ (or) $\rightarrow$ (implies) $\neg$ (not)
+	
Temporal Operators	$\Diamond$ (eventually) $\Box$ (always) $\mathcal{U}$ (until)

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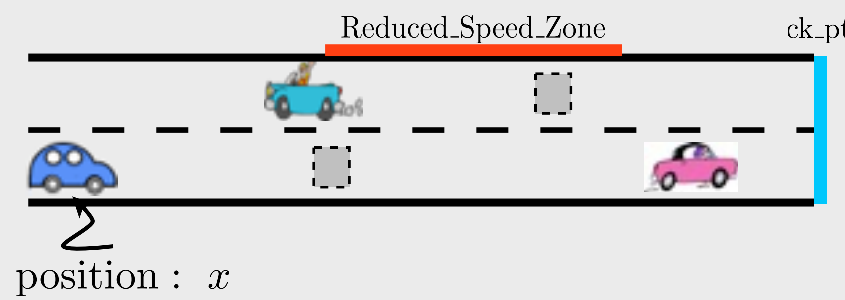

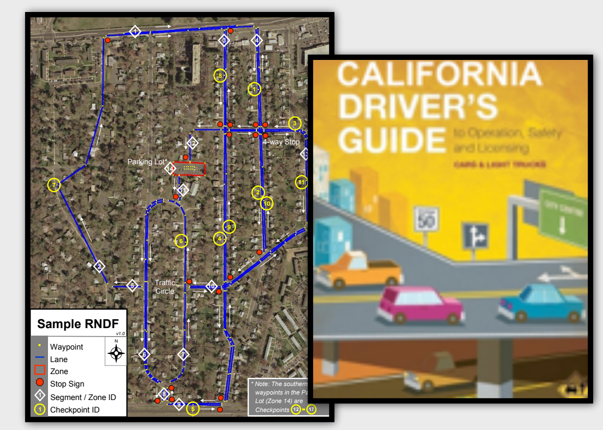

Traffic rules:

- No collision $\Box\,(\mathrm{dist}(x, \mathrm{Obs}) \geq X_{\mathrm{safe}} \wedge \mathrm{dist}(x, \mathrm{Loc}(\mathrm{Veh})) \geq X_{\mathrm{safe}})$
- Obey speed limits $\Box\,((x \in \mathrm{Reduced\_Speed\_Zone}) \rightarrow (v \leq v_{\mathrm{reduced}}))$
- Stay in travel lane unless blocked
- Intersection precedence & merging, stop line, passing,...

Goals:

- Eventually visit the check point $\Diamond(x = \mathrm{ck\_pt})$
- Every time check point is reached, eventually come to start $\Box((x = \mathrm{ck\_pt}) \rightarrow \Diamond(x = \mathrm{start}))$

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Propositional Logic

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Temporal

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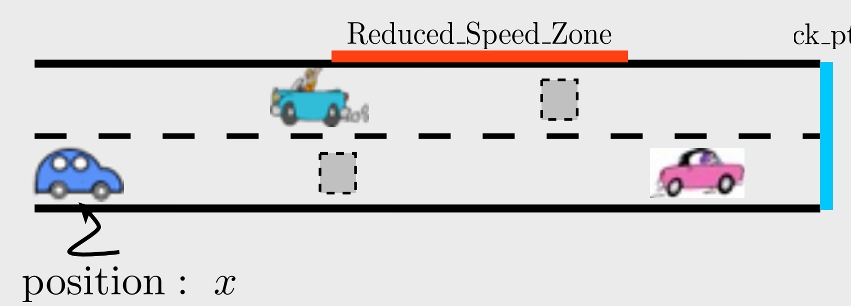

VMS_global.spc — Edited

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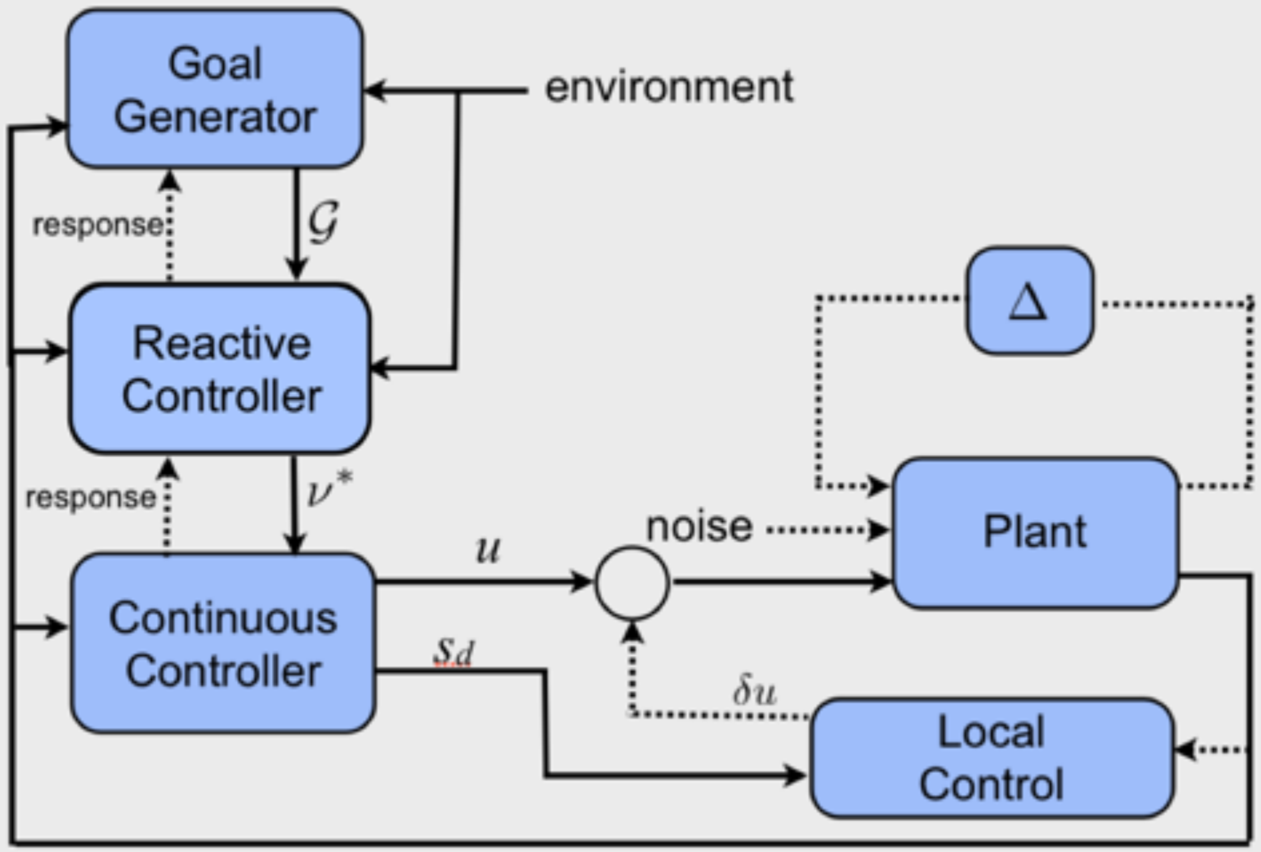
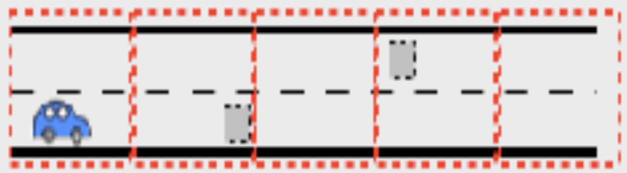
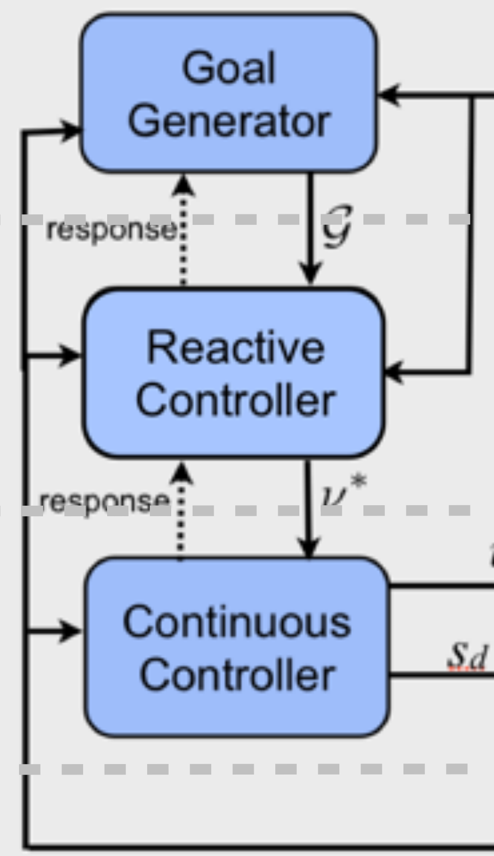
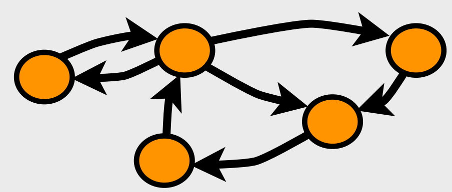
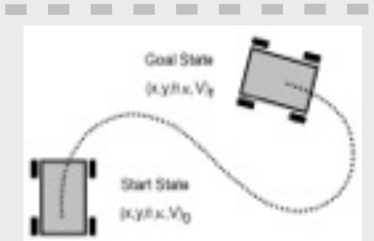
A widely explored approach



A widely explored approach

Different views		Multi-scale models	Synthesis method	Control protocol
	long-horizon specifications			
	short-horizon specifications			
	constraints on continuous state + input	$x_{t+1} = f(x_t, w_t, u_t)$ $x \in \mathcal{X}, u \in \mathcal{U}, w \in \mathcal{W}$		

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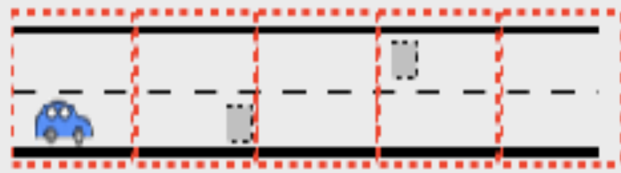
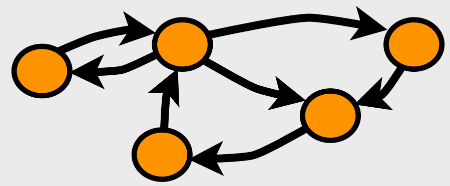
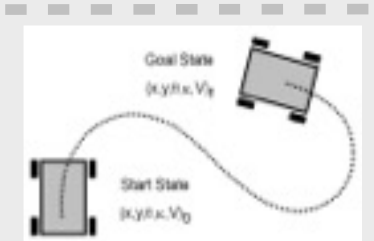
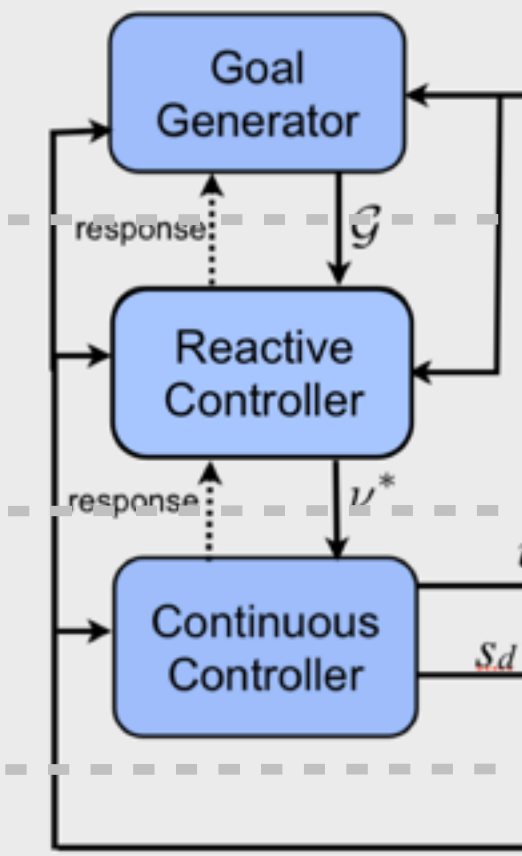
	long-horizon specifications		Iterative graph search
	short-horizon specifications		Two-player, turn-based graph game
	constraints on continuous state + input	$x_{t+1} = f(x_t, w_t, u_t)$ $x \in \mathcal{X}, u \in \mathcal{U}, w \in \mathcal{W}$	Constrained, finite-horizon optimal control

Control protocol



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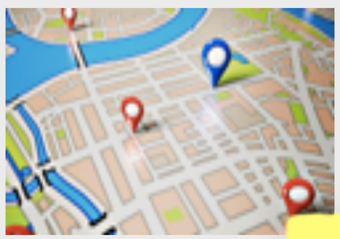
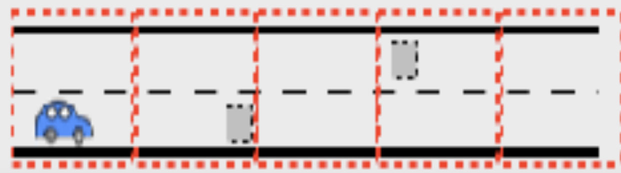
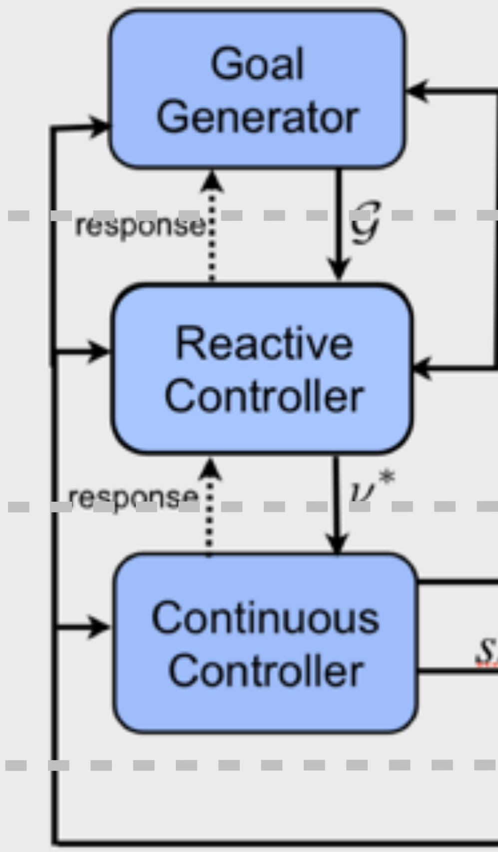
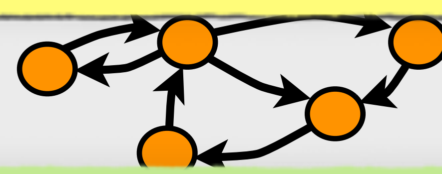
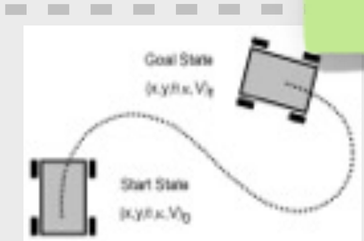
	long-horizon specifications		Iterative graph search	
	short-horizon specifications		Two-player, turn-based graph game	
	constraints on continuous state + input	$x_{t+1} = f(x_t, w_t, u_t)$ $x \in \mathcal{X}, u \in \mathcal{U}, w \in \mathcal{W}$	Constrained, finite-horizon optimal control	

Abstraction with "simulation" relation

(Finite-state) abstraction with "simulation" relation

Finite-state abstraction with "simulation" relations

$$x_{t+1} = f(x_t, w_t, u_t)$$
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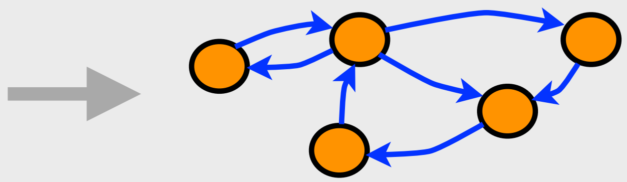

Every discrete transition can be "executed" under the continuous dynamics

Finite-state abstraction with "simulation" relations

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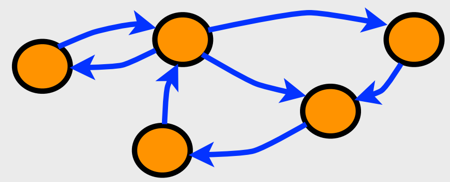

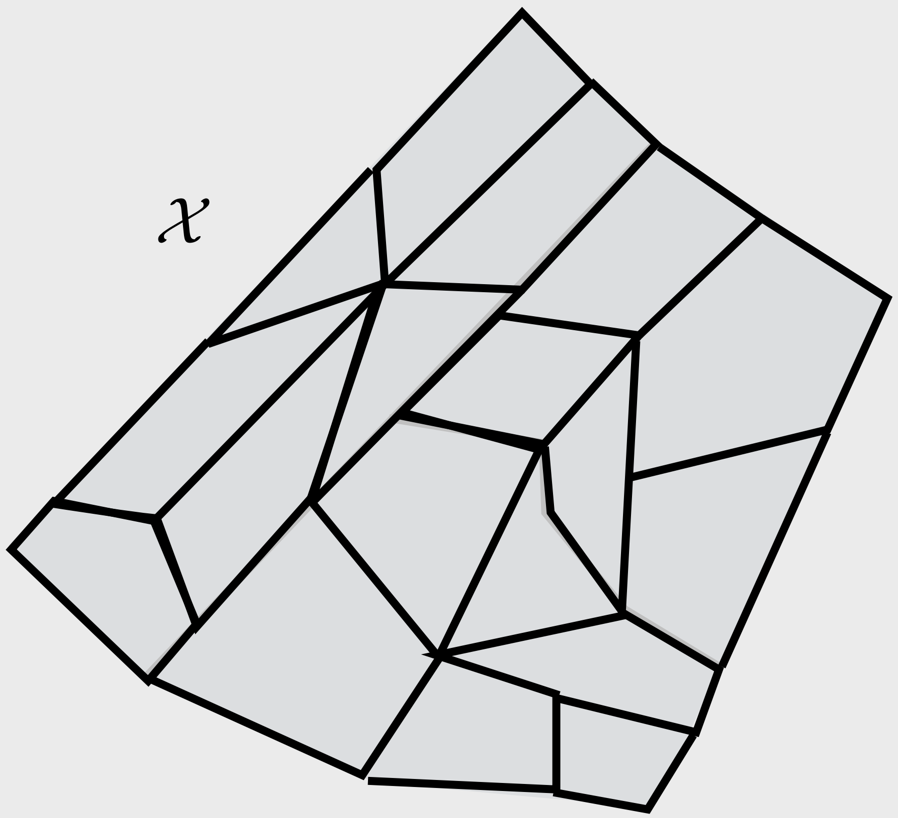

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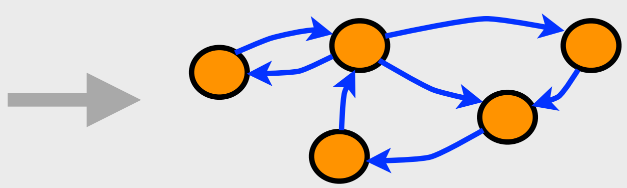

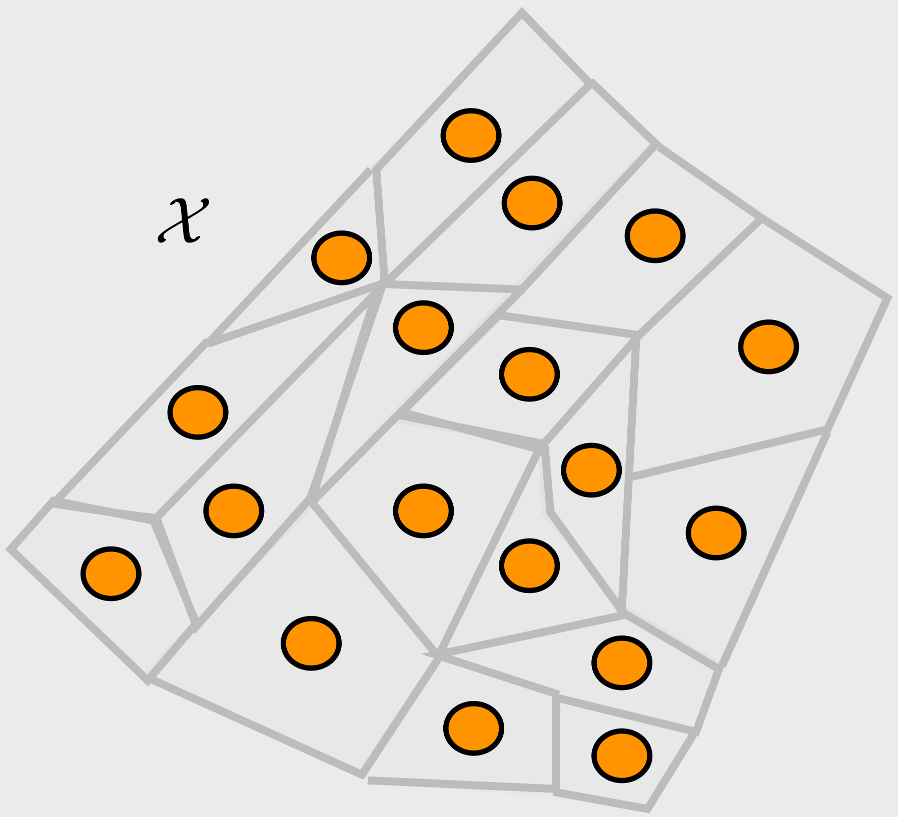

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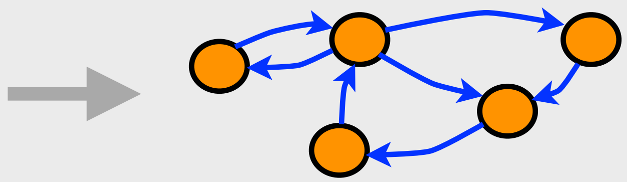

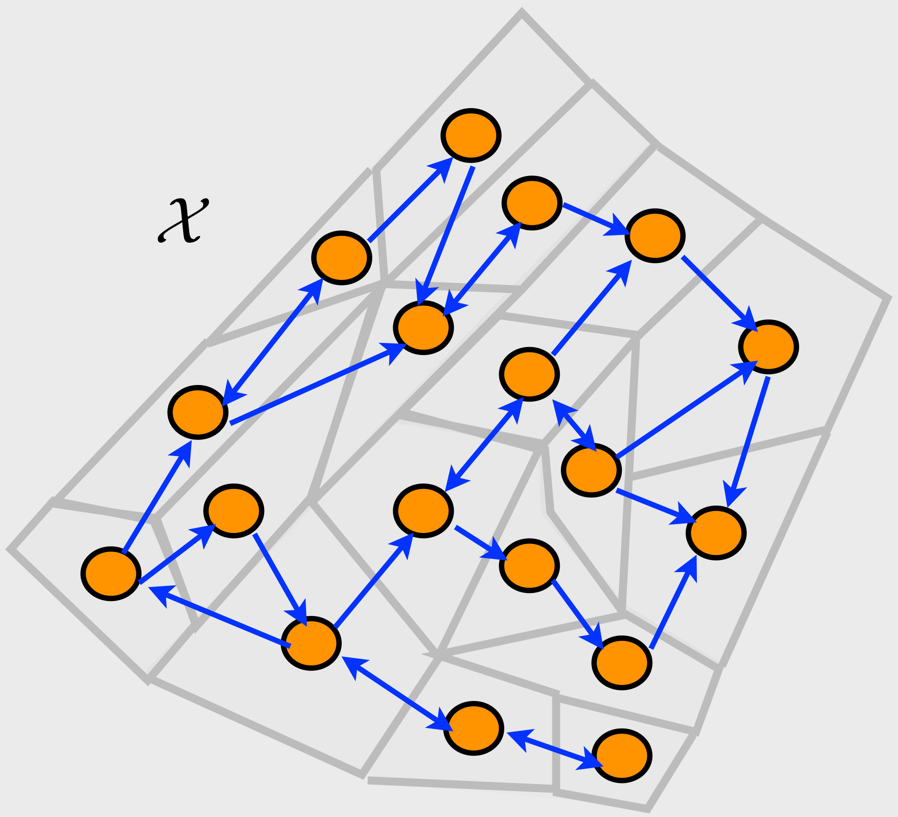

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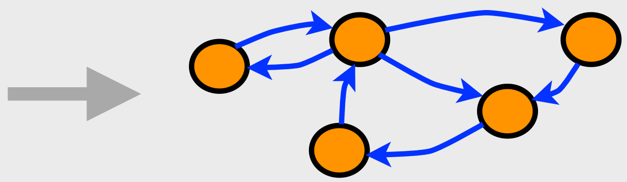

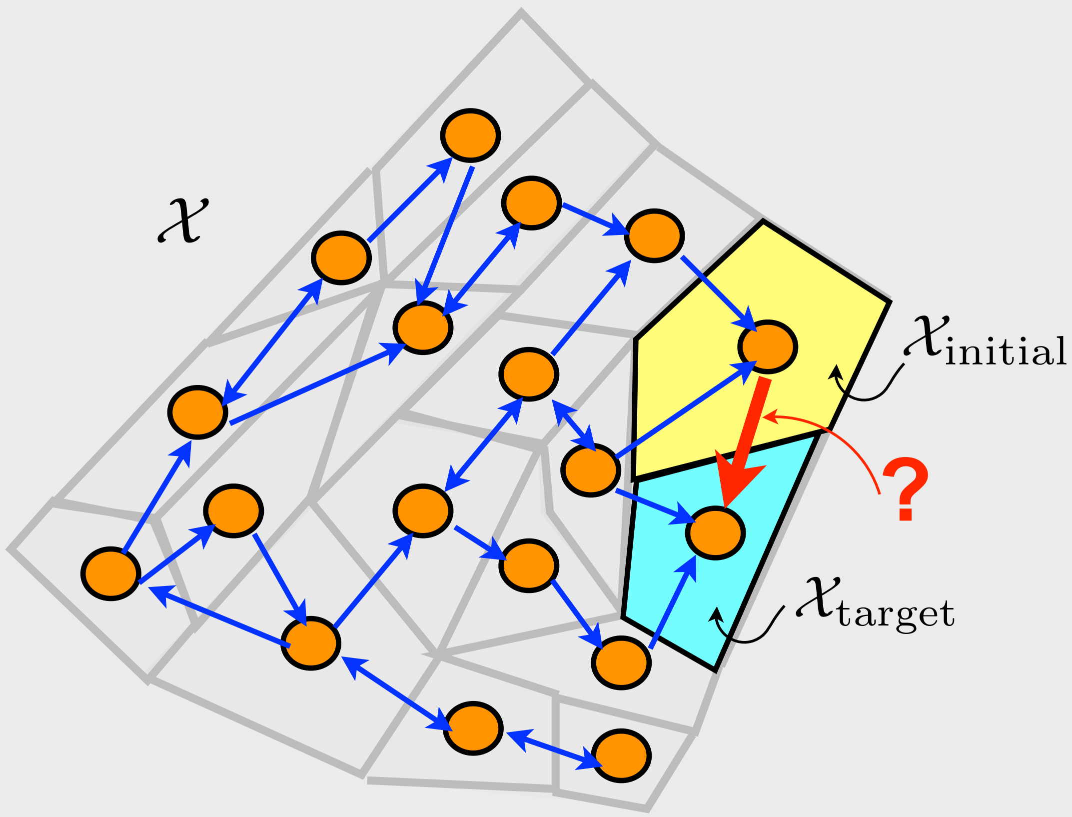

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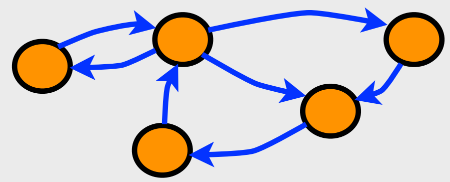
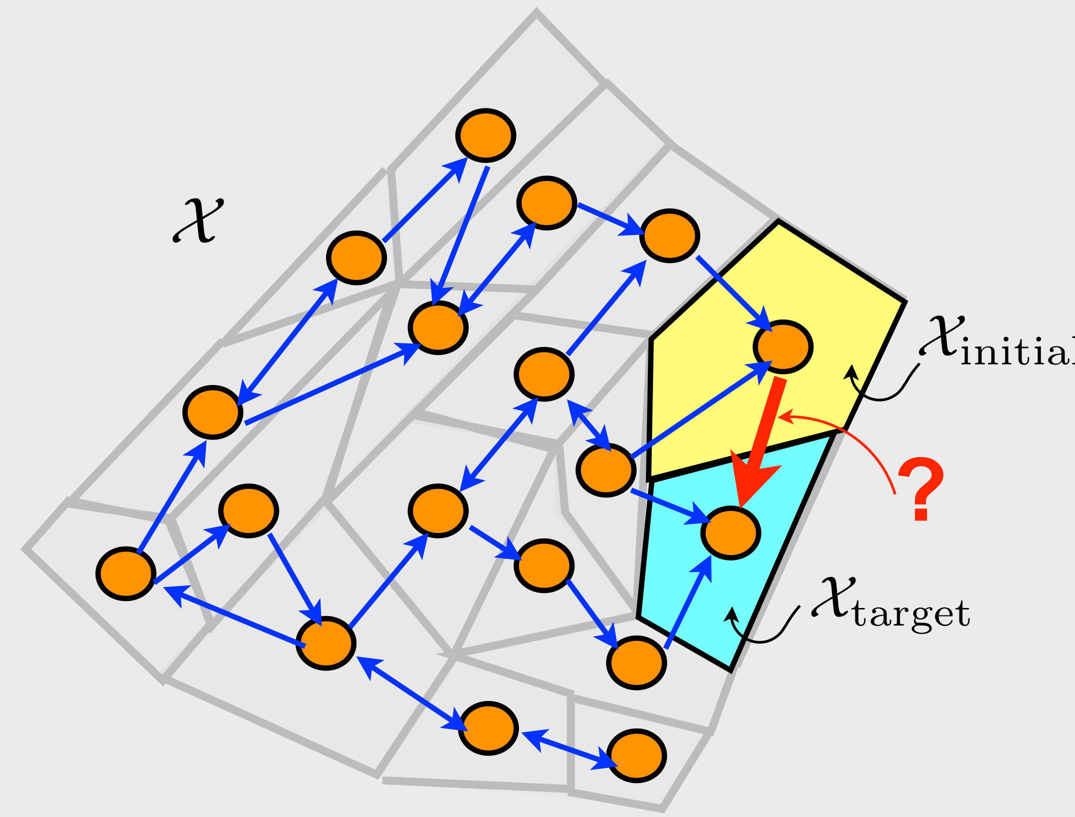
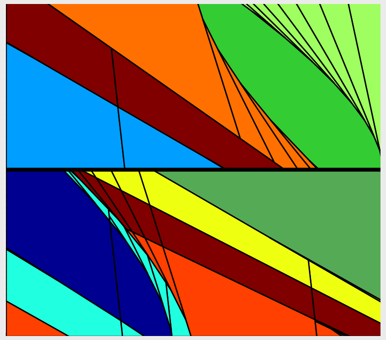




Every discrete transition can be “executed” under the continuous dynamics

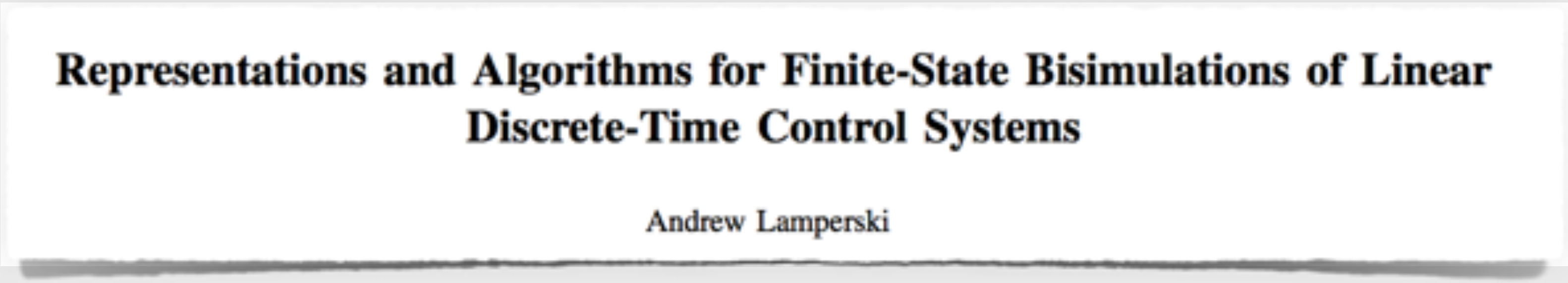
Why is discretization not necessarily a good idea?

Practically:
Complex partitions are needed.



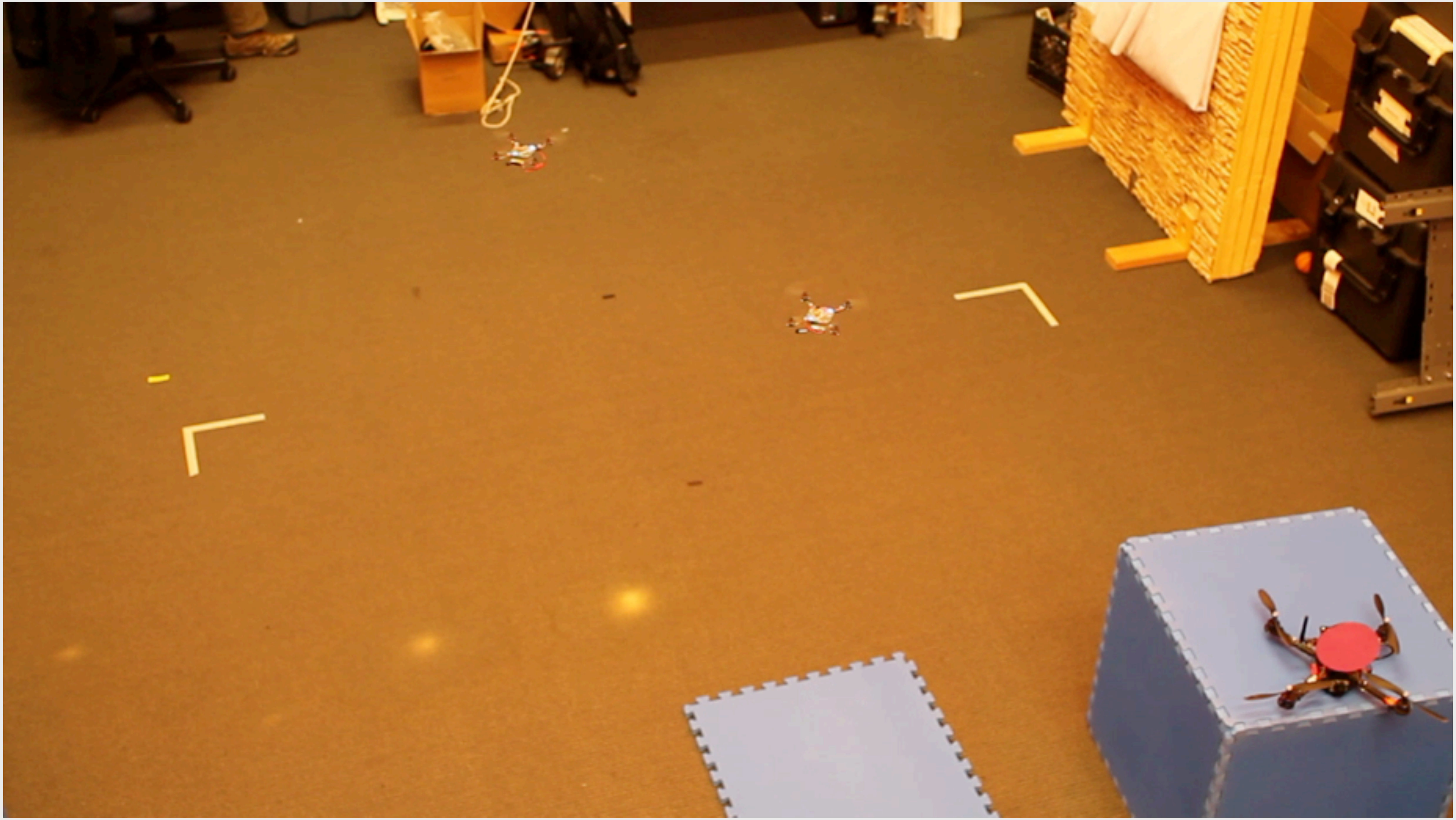
Theoretically:
Finite yet humongous discrete state spaces may be needed.

$$2^{2^{\cdot^{\cdot^{\cdot^{2^p}}}}}$$


Representations and Algorithms for Finite-State Bisimulations of Linear Discrete-Time Control Systems

Andrew Lamperski

Experiments



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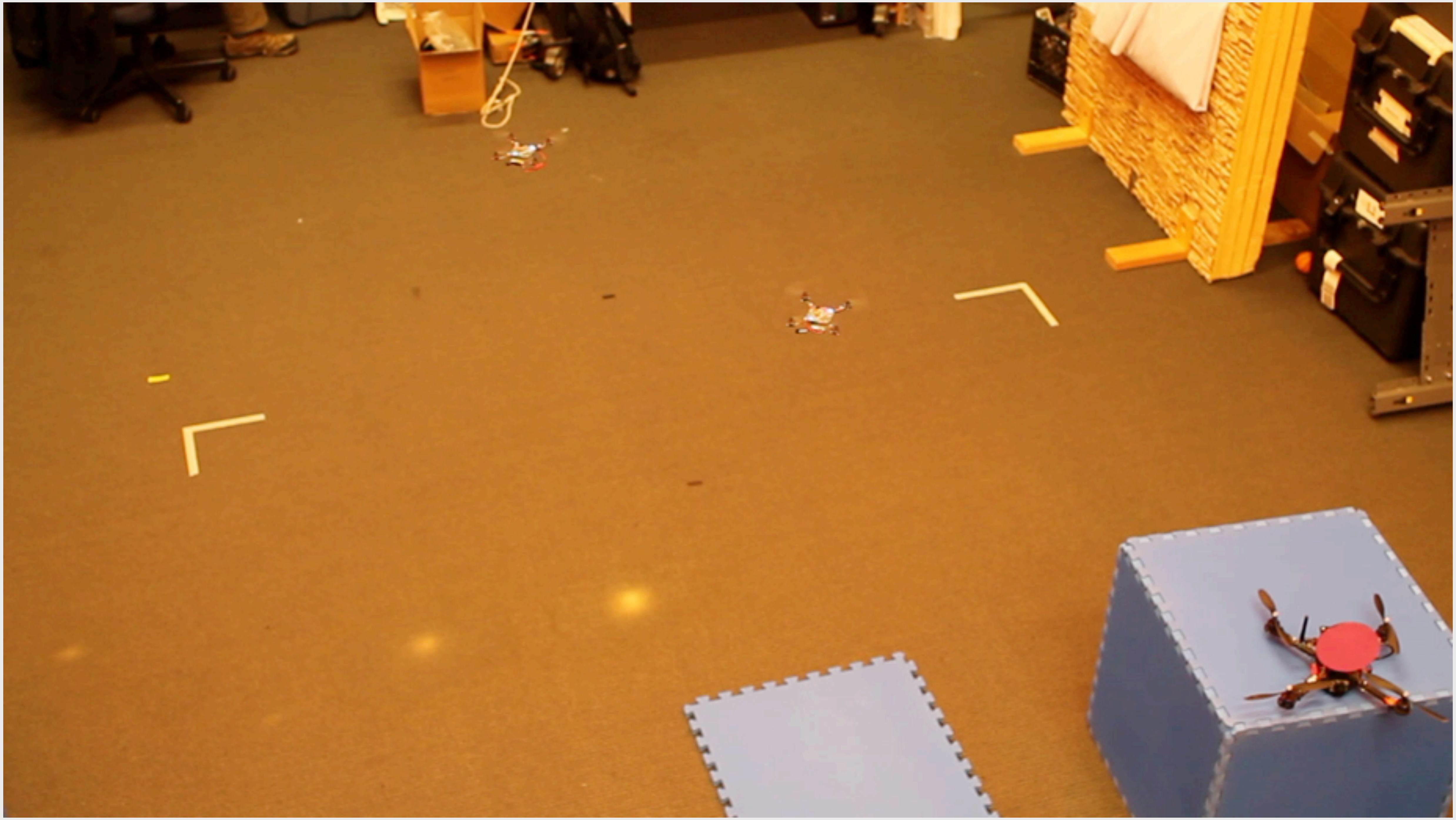

An alternative to explicit discretization:
no explicit discretization

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CDC 2016

Automata Theory Meets Approximate Dynamic Programming: Optimal Control with Temporal Logic Constraints

Ivan Papusha[†] Jie Fu[*] Ufuk Topcu[‡] Richard M. Murray[†]

An alternative to explicit discretization:
no explicit discretization

CDC 2016

Automata Theory Meets Approximate Dynamic Programming: Optimal Control with Temporal Logic Constraints

Ivan Papusha[†] Jie Fu[*] Ufuk Topcu[‡] Richard M. Murray[†]

TAC 2015

Automata Theory Meets Barrier Certificates: Temporal Logic Verification of Nonlinear Systems

Tichakorn Wongpiromsarn[*] Ufuk Topcu[†] Andrew Lamperski[‡]

Problem statement

Given

System model

$$\dot{x} = f(x, u), \quad x(0) = x_0$$

$$x(t) \in \mathcal{X} \subseteq \mathbb{R}^n, \; u(t) \in \mathcal{U} \subseteq \mathbb{R}^m$$

continuous time, continuous state
with assumptions on f for existence,
uniqueness and Zeno-freeness of solutions

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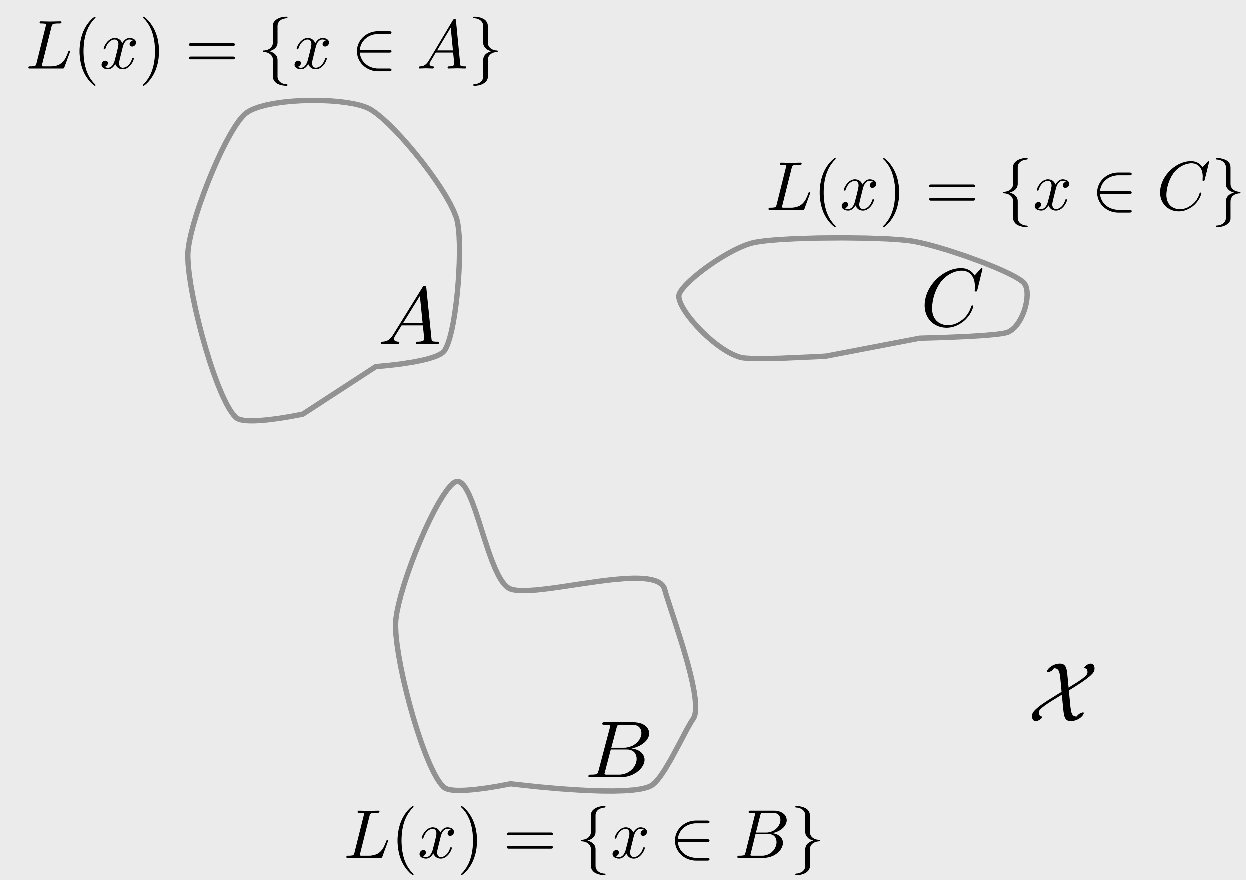

Labeling function $\quad L : \mathcal{X} \to \Sigma = 2^{\mathcal{AP}}$

(what properties hold at a given state?)

Problem statement

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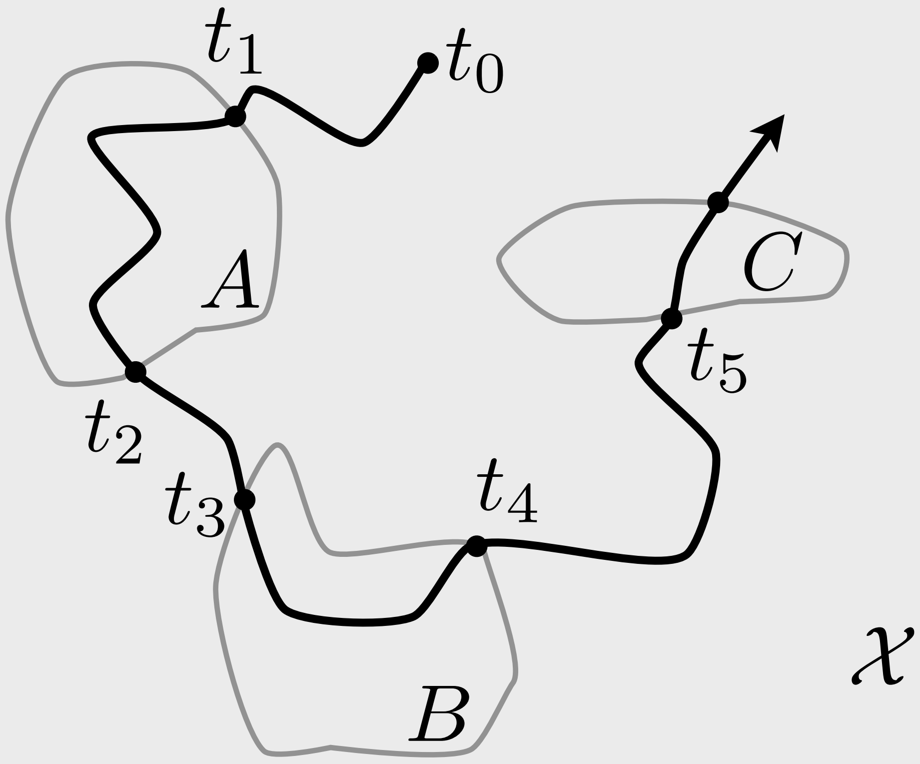
System model

$$\dot{x} = f(x, u), \quad x(0) = x_0$$

$$x(t) \in \mathcal{X} \subseteq \mathbb{R}^n, \ u(t) \in \mathcal{U} \subseteq \mathbb{R}^m$$



Labeling function $L : \mathcal{X} \to \Sigma = 2^{\mathcal{AP}}$

(what properties hold at a given state?)

$$0 = t_0 < t_1 < \cdots < t_N = T$$

$$L(x(t)) = L(x(t_k)), t_k \leq t < t_{k+1}$$

$$L(x(t_k^-)) \neq L(x(t_k^+))$$

Problem statement

Given

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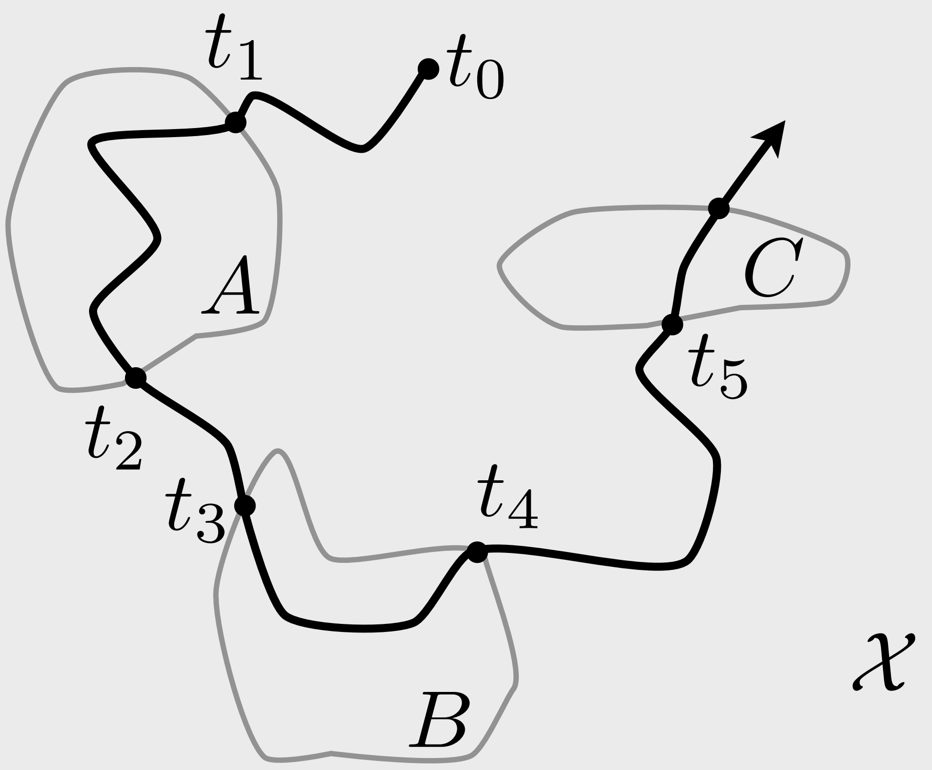

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“discrete” behavior: $\mathbb{B}(\phi(x_0, [0,T], u)) = \sigma_0 \sigma_1 \ldots \sigma_{N-1} \in \Sigma^*$

with $\sigma_k = L(x(t_k))$

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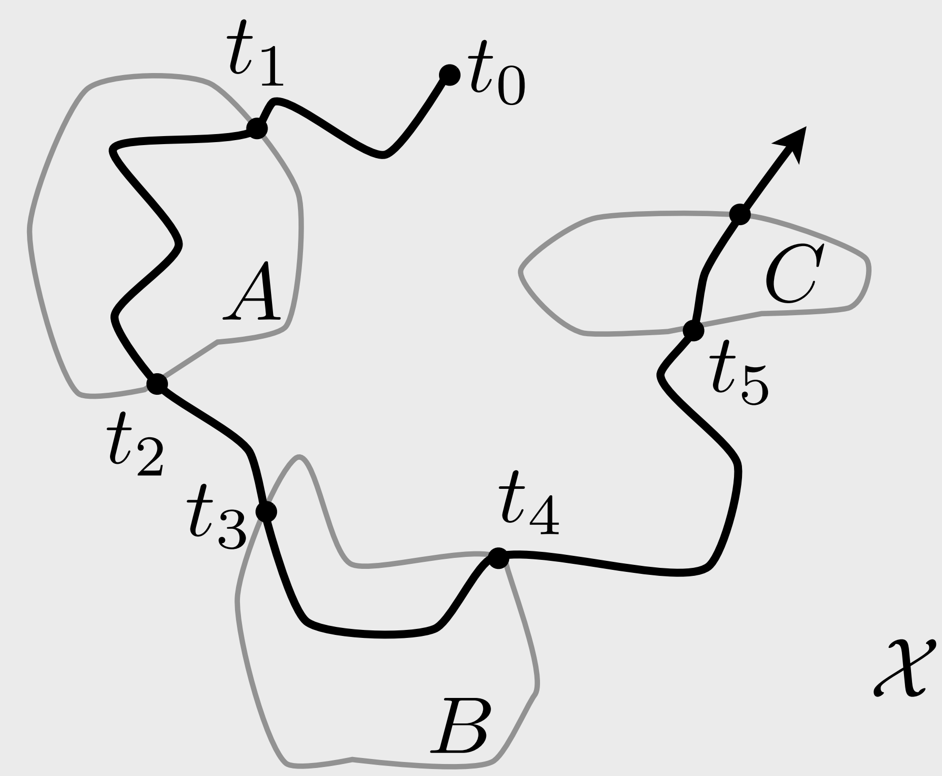
(what properties hold at a given state?)

Co-safe temporal logic specification φ

(every satisfying word has a finite “good” prefix)

A final state $\quad x_f \in \mathcal{X}$ and a final time T.



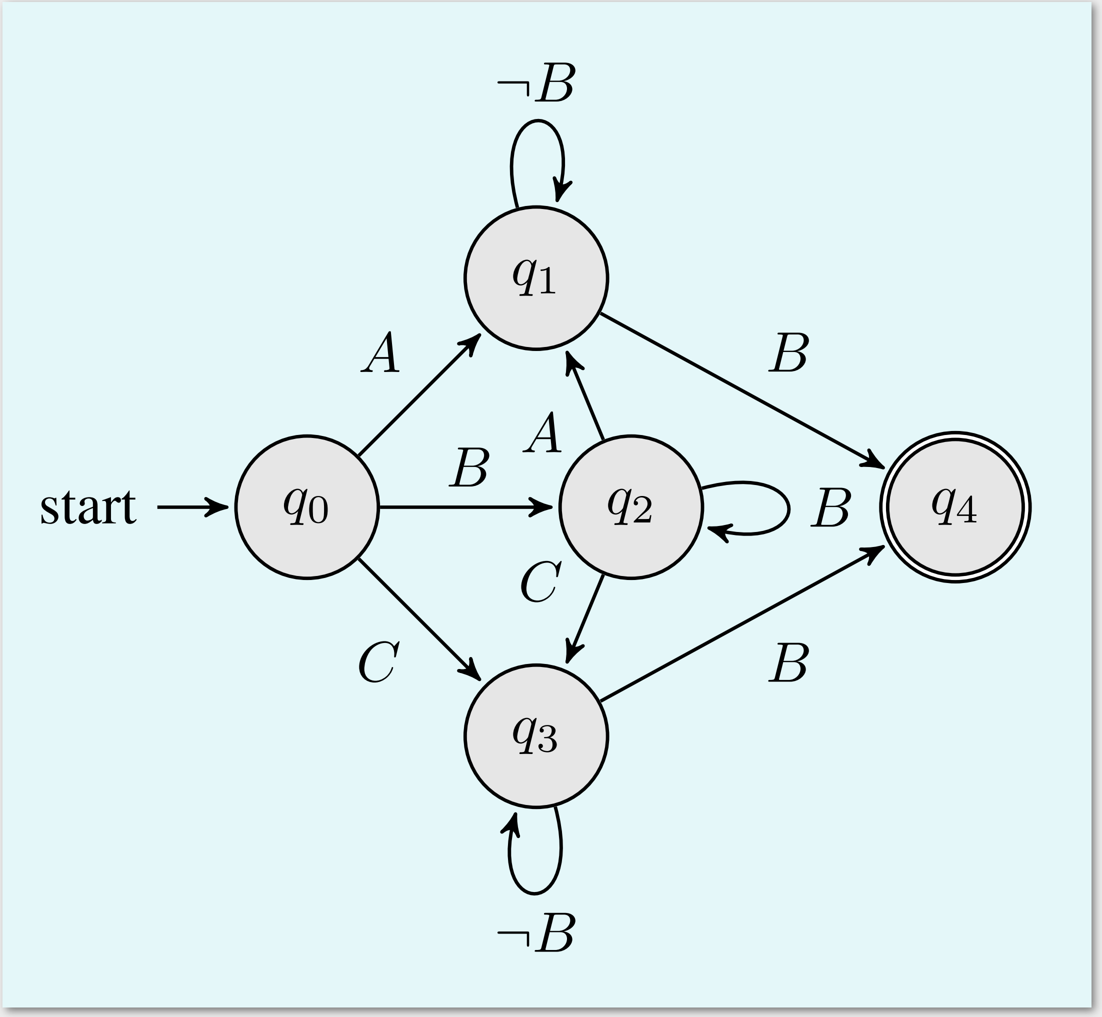
$$\mathbb{B}(\phi(x_0, [0, T], u)) = \sigma_0 \sigma_1 \ldots \sigma_{N-1} \in \Sigma^*$$

with $\sigma_k = L(x(t_k))$

De-tour: Automaton representation for temporal logic

$$(A \rightarrow \Diamond B) \wedge (C \rightarrow \Diamond B) \wedge (\Diamond A \vee \Diamond C)$$

Machine-interpretable representation of all words that satisfy the corresponding temporal logic formula



Deterministic finite automata are sufficient for co-safe linear temporal logic formulas

Problem statement (2)

Model

$$\dot{x} = f(x, u), \quad x(0) = x_0$$

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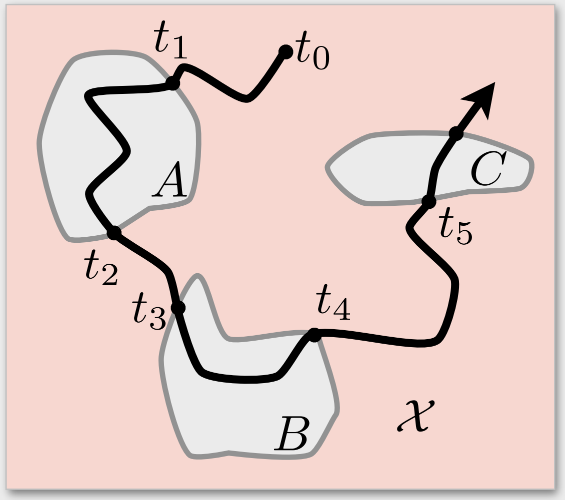

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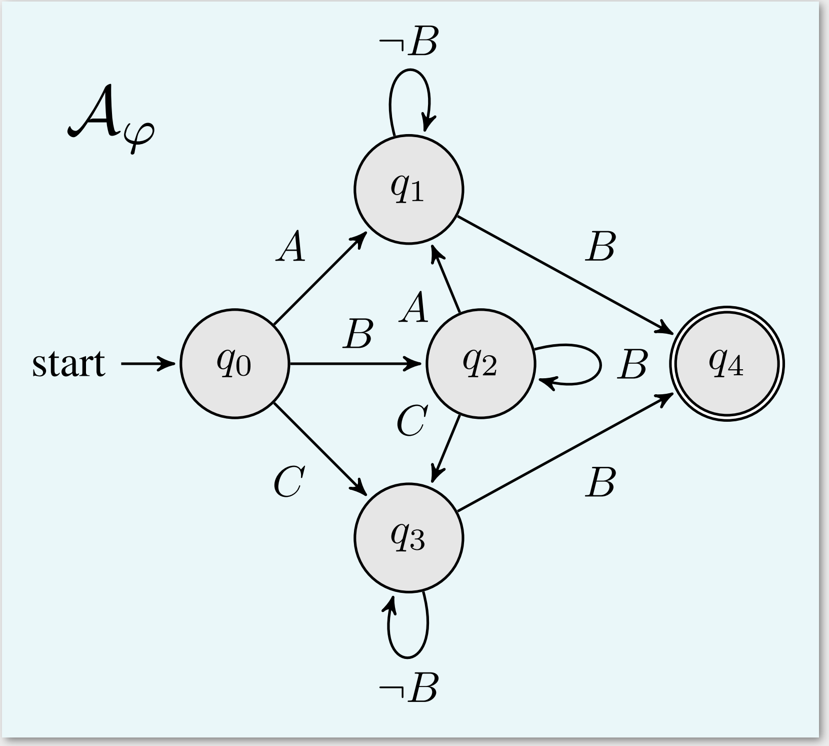
Specification φ



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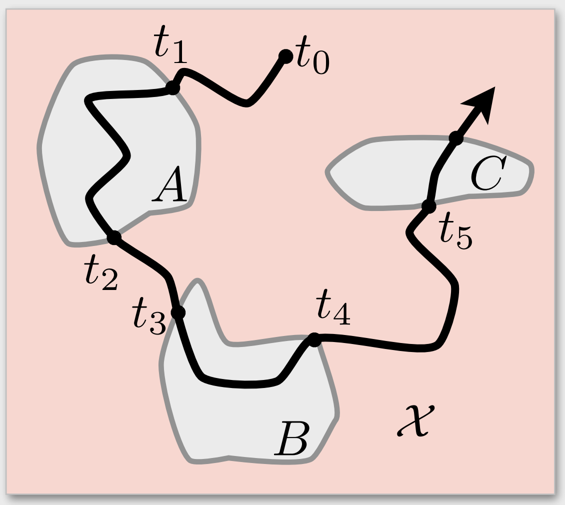

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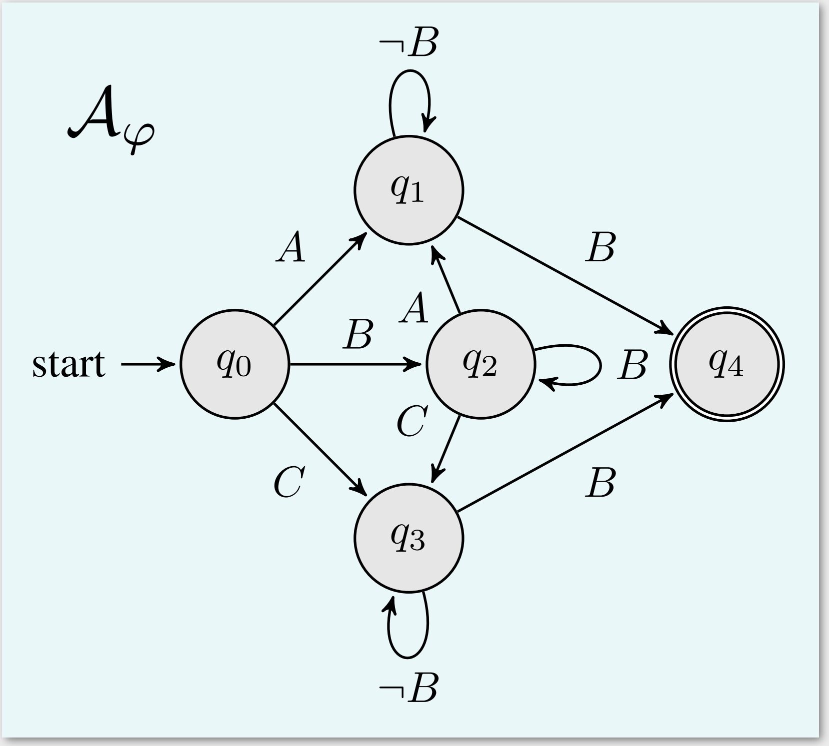

Compute a control law u that minimizes

$$\int_0^T \ell(x(\tau), u(\tau))\, d\tau + \sum_{k=0}^{N} s(x(t_k), q(t_k^-), q(t_k^+))$$

l: loss function

s: cost of mode transition

subject to $x(T) = x_f$ and

$$\mathbb{B}(\phi(x_0, [0, T], u)) \in \mathcal{L}(\mathcal{A}_\varphi).$$

all discrete behavior satisfies the specification

Related work

$$\int_0^T \ell(x(\tau), u(\tau))\, d\tau$$

$$\dot{x} = f(x, u), \quad x(0) = x_0$$
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Temporal logic specification

$$(A \rightarrow \Diamond B) \wedge (C \rightarrow \Diamond B) \wedge (\Diamond A \vee \Diamond C)$$

restrict to simple specifications

make it a formal methods problem

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Hedlund & Rantzer
(optimal control for hybrid systems
+ convex dynamic programming)

Xu & Antsaklis
(optimal control for switched systems)

Kariotoglou, et al.
(approximate dynamic programming
for stochastic reachability)

make it a formal methods problem

Habets & Belta

Wongpiromsarn, et al.

Wolff, et al.

Fainekos, et al.

Product hybrid system

The problem can be formulated as a dynamic programming problem over a **product hybrid system**:

$$\langle\ Q,\ \mathcal{X},\ E,\ f,\ R,\ G\ \rangle$$

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The problem can be formulated as a dynamic programming problem over a **product hybrid system**:

set of continuous states

continuous vector field

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set of states of the specification automaton

$E \subseteq Q \times \Sigma \times Q$

set of discrete transitions

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- The continuous state x evolves according to the vector field.
- The evolution of the discrete state q is governed by the automaton.
- A discrete transition is triggered when x crosses a boundary between two labeled regions.

Product hybrid system

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set of states of the specification automaton

$E \subseteq Q \times \Sigma \times Q$

set of discrete transitions

$R = \{R_q \mid q \in Q\}$

R_q refers to x reachable while the automaton is in or transitions to mode q

a collection of guard regions in X:
For each q, x evolves inside R_q until it enters a guard region $G_{(q,\sigma,q')}$ and a transition to q' is made

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Dynamic programming formulation

Hybrid Hamilton-Jacobi-Bellman equations over the product space

V*: optimal cost-to-go subject to the specifications

$$0 = \min_{u \in \mathcal{U}} \left\{ \frac{\partial V^\star(x,q)}{\partial x} \cdot f(x,u) + \ell(x,u) \right\}$$

$$\forall x \in R_q, \ \forall q \in Q$$

$$V^\star(x,q) = \min_{q'} \{ V^\star(x,q') + s(x,q,q') \}$$

$$\forall x \in G_e, \ \forall e = (q, \sigma, q') \in E$$

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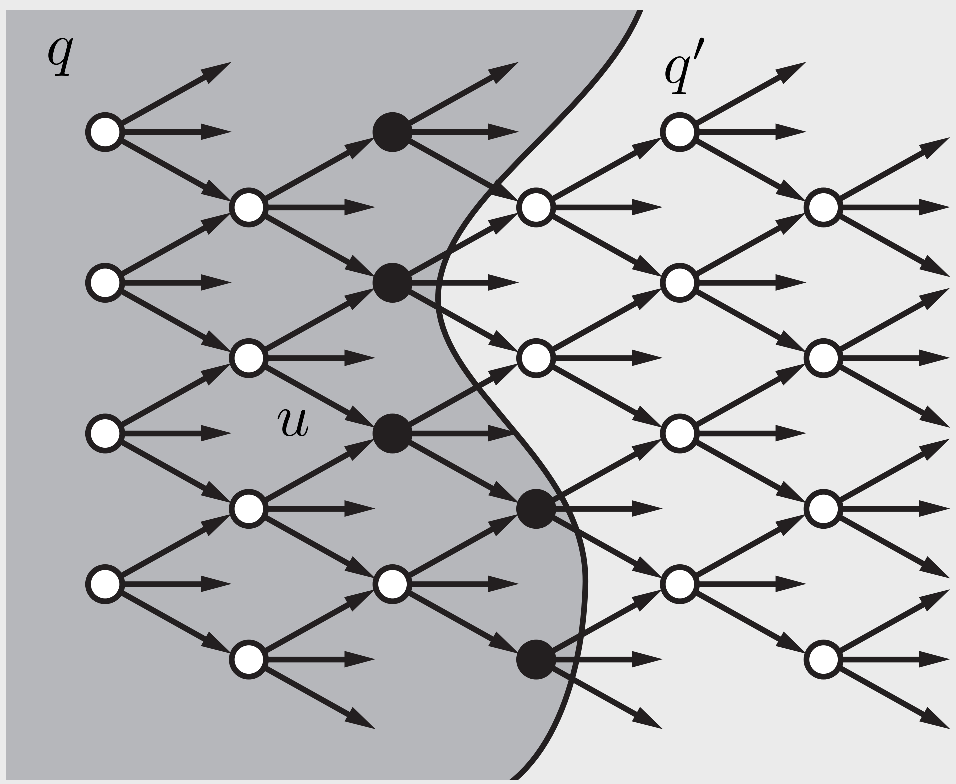

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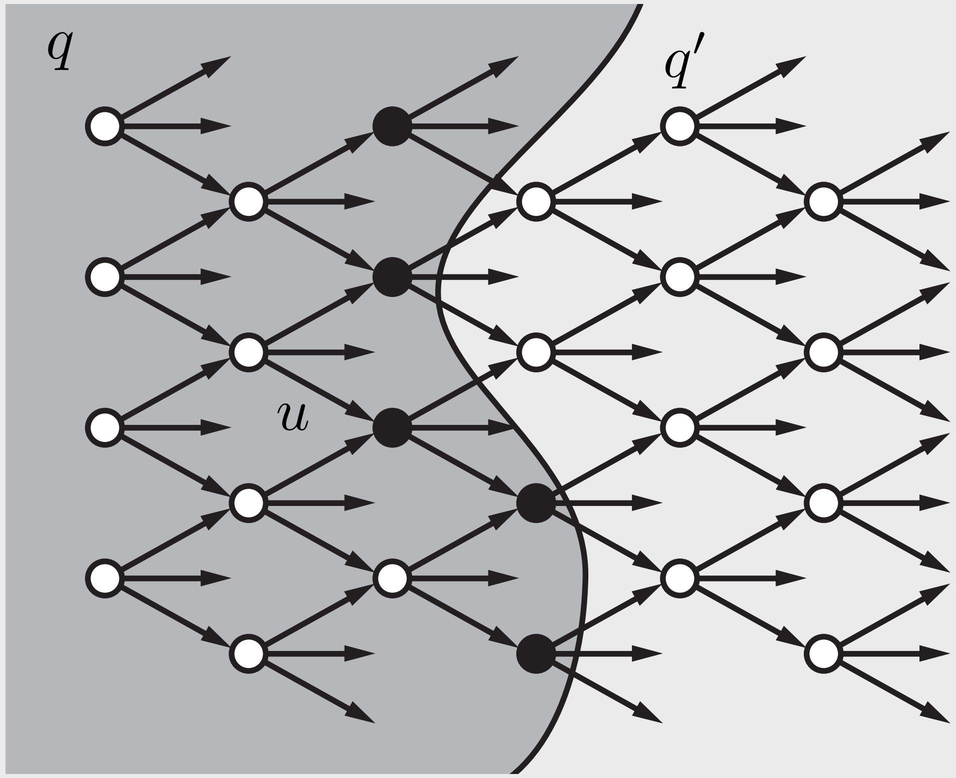

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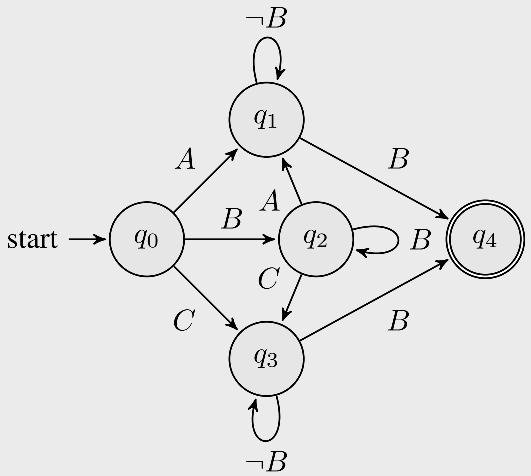

At the "terminal" state:

$$0 = V^\star(x_f, q_f), \quad \forall q_f \in F$$

(Toward computable) lower bounds on the optimal cost

$$0 \leq \frac{\partial V(x,q)}{\partial x} \cdot f(x,u) + \ell(x,u) \qquad \forall x \in R_q,\ \forall u \in \mathcal{U},\ \forall q \in Q$$

$$0 \leq V(x,q') - V(x,q) + s(x,q,q') \qquad \forall x \in G_e,\ \forall e = (q,\sigma,q') \in E$$

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***V*: approximate value function**

A function V that satisfies the above conditions is an under-estimator for the optimal value function V*:

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Intuition from purely discrete version:

$$V^* = \mathbb{T}V^*$$

$$V \leq \mathbb{T}V \Rightarrow V \leq V^*$$

Approximate value function and approximately optimal control law

Parametrize V with pre-specified basis functions ϕ:

$$V(x,q) = \sum_{i=1}^{n_q} w_{i,q}\phi_{i,q}(x)$$

basis:
function of x,
indexed by q

Search for approximate value function that maximizes $V(x_0,q_0)$.

(one of the many scalarizations)

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(one of the many scalarizations)

Given V, an approximately optimal control law:

$$u(x,q) = \arg\min_{u\in\mathcal{U}} \left\{ \frac{\partial V(x,q)}{\partial x} \cdot f(x,u) + \ell(x,u) \right\}$$

Mode switchings are autonomous, driven by the evolution of x.

Search for approximate value function

Linear system: $\dot{x}(t) = Ax(t) + Bu(t), \quad x(0) = x_0,$

Quadratic continuous cost: $\ell(x, u) = x^T Q x + u^T R u, \quad Q \succeq 0, \quad R \succ 0$

Constant switching cost: $s(x, q, q') = \xi \cdot \mathbb{I}\left(\{(q, q') \mid q \neq q'\}\right)$

For each $q \in Q$, parametrize V by P_q, r_q, t_q: $V(x, q) = x^T P_q x + 2 r_q^T x + t_q$

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$$0 \leq \begin{bmatrix} x \\ u \\ 1 \end{bmatrix}^T \begin{bmatrix} A^T P_q + P_q A + Q & P_q B & A^T r_q \\ B^T P_q & R & B^T r_q \\ r_q^T A & r_q^T B & 0 \end{bmatrix} \begin{bmatrix} x \\ u \\ 1 \end{bmatrix} \quad \forall x \in R_q, \ \forall u \in \mathcal{U}, \ \forall q \in Q$$

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semi-infinite optimization problem

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Solving the semi-infinite optimization problem

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For *quadratically representable* R_q, G_e and U,

(1) use the S-procedure to resort to finite sufficient conditions for the semi-infinite constraints

(2) translate into a semidefinite program

Solving the semi-infinite optimization problem

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"S-procedure"

$$M_0, M_1 : \mathbb{R}^n \to \mathbb{R}$$

$$\boxed{M_1 \geq 0 \Rightarrow M_0 \geq 0}$$

$$\Uparrow$$

$$\boxed{\begin{array}{c} \exists \lambda \geq 0 \text{ s.t.} \\ M_0(\zeta) - \lambda M_1(\zeta) \geq 0 \ \forall \zeta \end{array}}$$

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Are R_q and G_e quadratically representable?

- Can be decided based on the atomic propositions in the specification.

Example

Linear quadratic system

$$A = \begin{bmatrix} 2 & -2 \\ 1 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 1 \\ 1 \end{bmatrix},$$

$$Q = I, \quad R = 1, \quad \xi = 1,$$

$$x_f = (0, 0)$$

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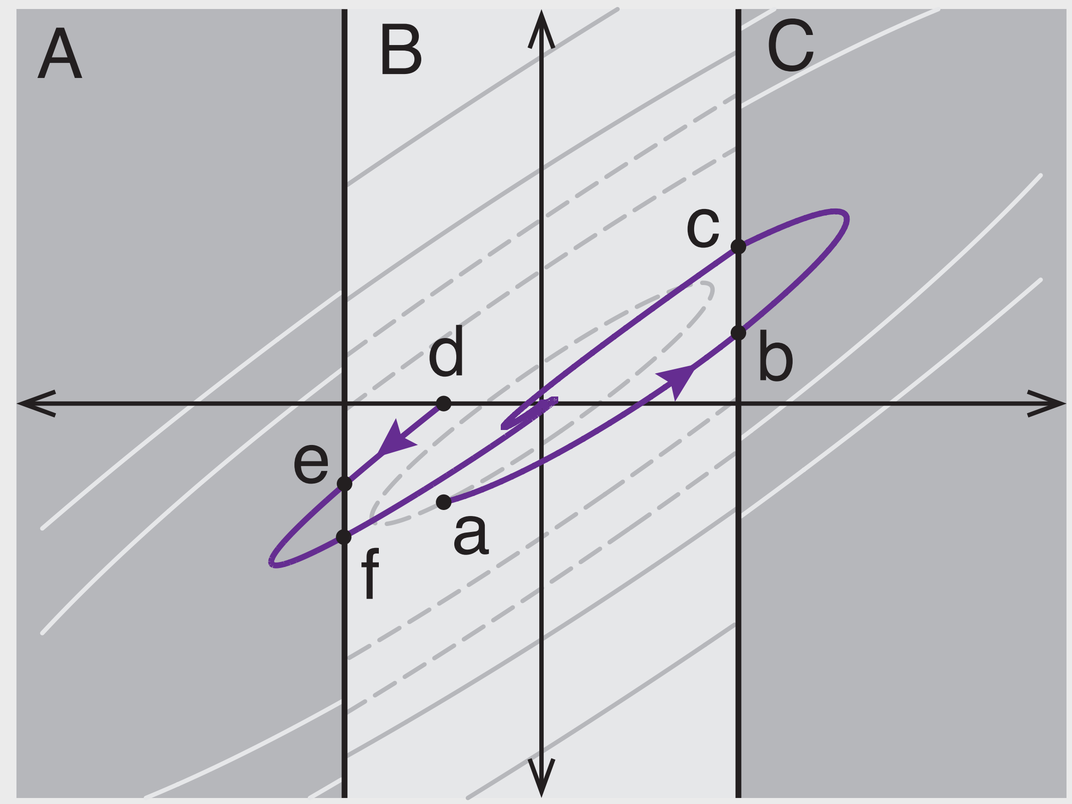

Specification

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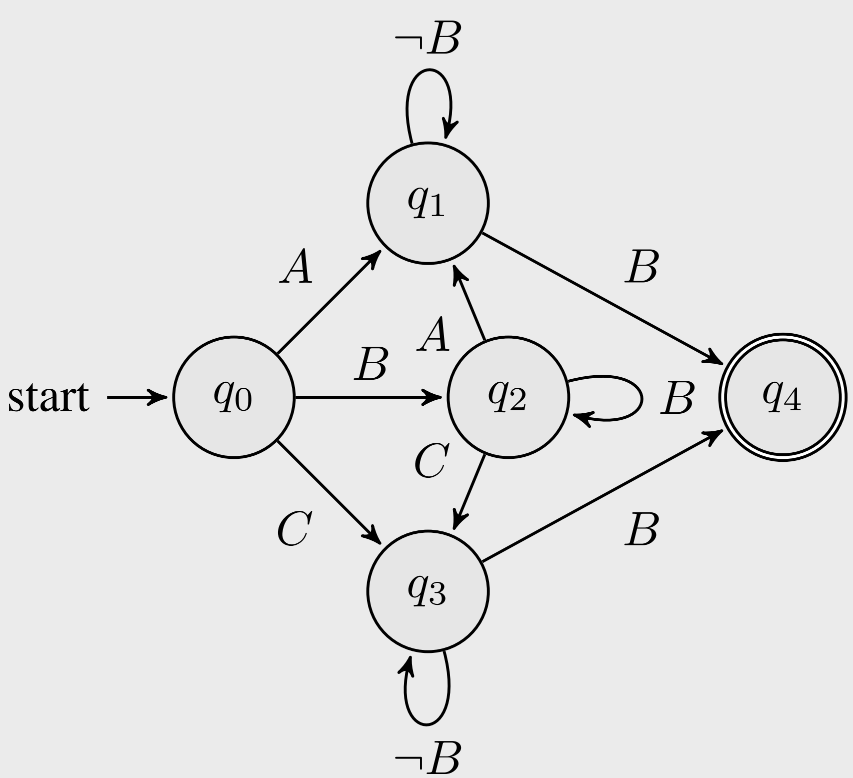

Example

Linear quadratic system

$$A = \begin{bmatrix} 2 & -2 \\ 1 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 1 \\ 1 \end{bmatrix},$$

$$Q = I, \quad R = 1, \quad \xi = 1,$$

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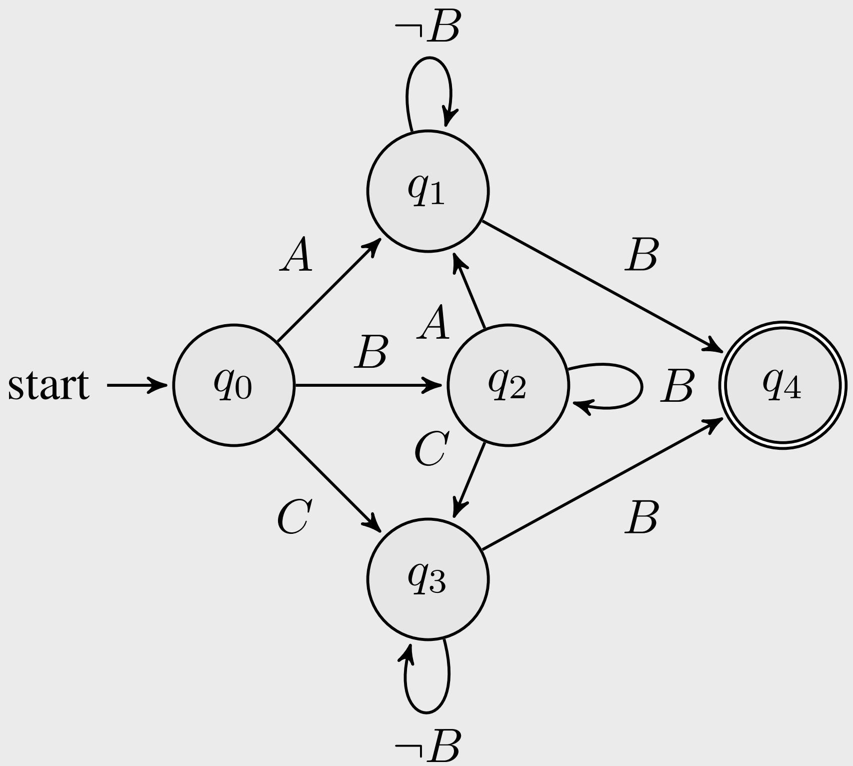

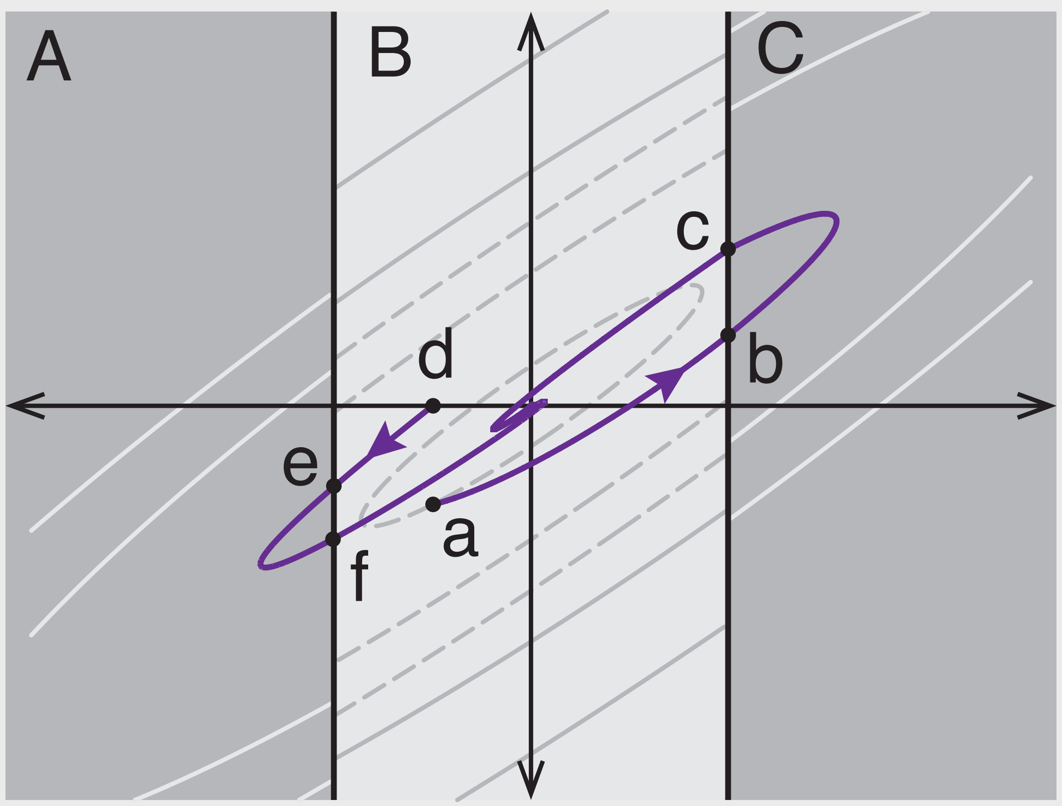

Compare the spectra of the closed-loop matrix in different modes

$$A_q^{\mathrm{cl}} = A - BR^{-1}B^T P_q^\star$$

$$\lambda(A_{q_0}^{\mathrm{cl}}) = \{0.786 \pm 1.144i\}$$

$$\lambda(A_{q_4}^{\mathrm{cl}}) = \{-1 \pm i\}$$

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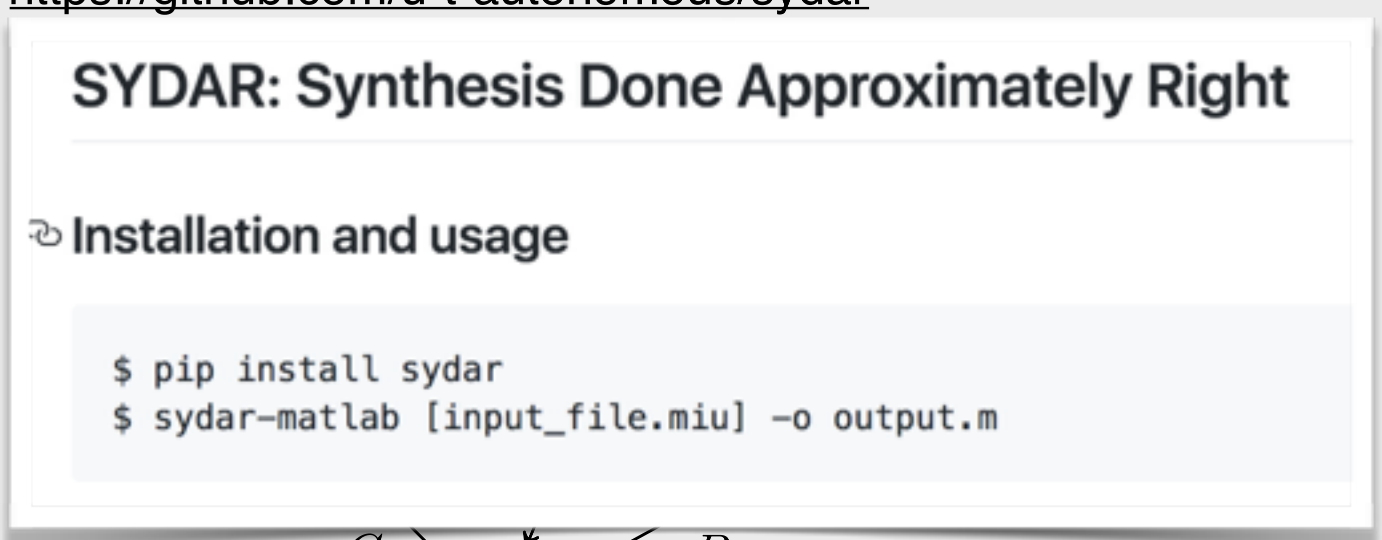
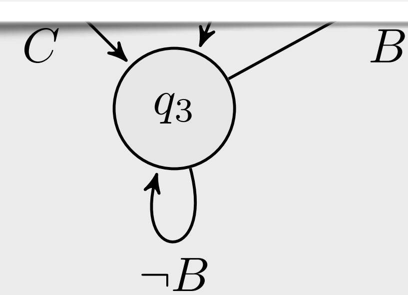
https://github.com/u-t-autonomous/sydar

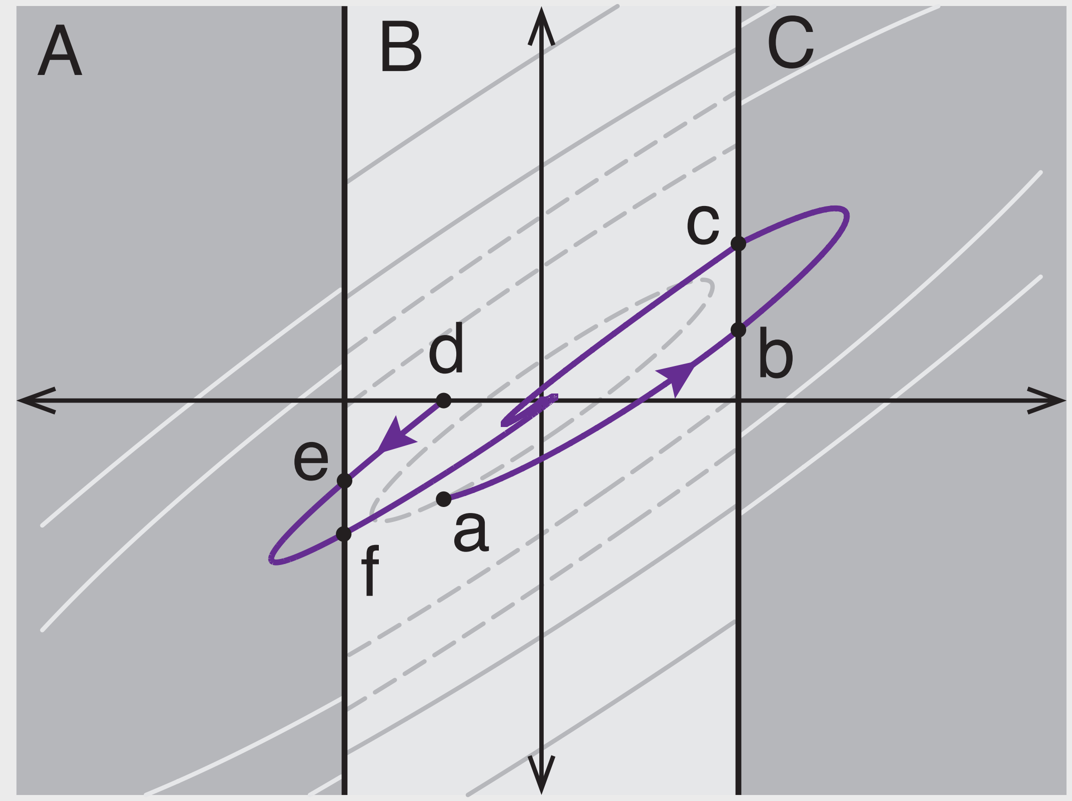


SYDAR: Synthesis Done Approximately Right

Installation and usage

```
$ pip install sydar
$ sydar-matlab [input_file.miu] -o output.m
```





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No need for explicit finite abstraction
(w.r.t. the dynamics)

No need for expensive reachability
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Hope for scalability?

Scalability goal:
"Can we synthesize temporal-logic-constrained controllers for systems with **50 continuous states**?"

$$0 \leq \begin{bmatrix} x \\ u \\ 1 \end{bmatrix}^T \begin{bmatrix} A^T P_q + P_q A + Q & P_q B & A^T r_q \\ B^T P_q & R & B^T r_q \\ r_q^T A & r_q^T B & 0 \end{bmatrix} \begin{bmatrix} x \\ u \\ 1 \end{bmatrix}$$

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Conservatism — S-procedure and basis selection

Policy is approximately optimal (bounds on sub optimality possible!)

Only co-safe temporal logic specifications (at this point)

What is next?

usual suspects

Demonstrate scalability

Reduce conservatism

Extend to broader classes dynamics — hybrid, nonlinear,…

Expand the family of specifications

new opportunities

Open up a broad set of new problems to ideas from controls and optimization

Automata Theory Meets Approximate Dynamic Programming: Optimal Control with Temporal Logic Constraints

Ivan Papusha† Jie Fu* Ufuk Topcu‡ Richard

Automata Theory Meets Barrier Certificates: Temporal Logic Verification of Nonlinear Systems

Tichakorn Wongpiromsarn* Ufuk Topcu† Andrew Lamperski‡

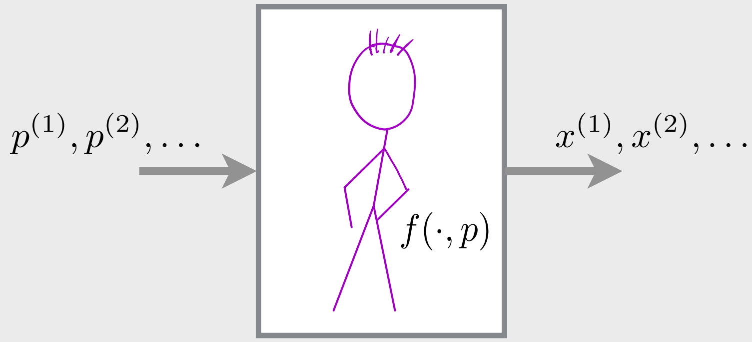
Learning from expert demonstrations

- Incorporating **expert demonstrations** into an autonomous system is difficult
- Even when expert demonstrations are somehow incorporated, generalizing to unseen scenarios can be **unsafe**.
- Can we generalize in a “safe” way using side information?
  - what does “safe” mean?
  - what kind of “side information” can be incorporated?

Learning by expert demonstration

Learning by expert demonstration

1. Expert gives "optimal" demonstrations



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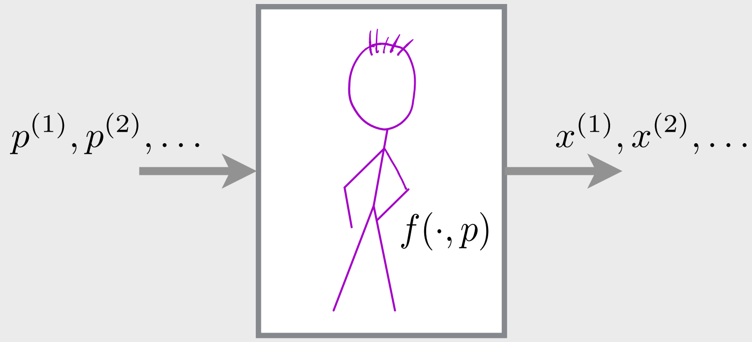
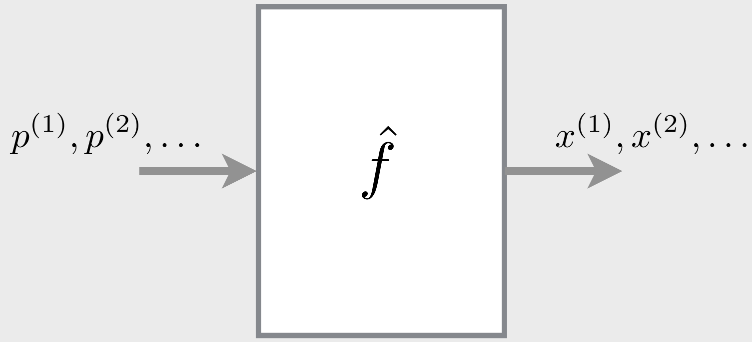


2. Demonstrations used to "learn" the expert



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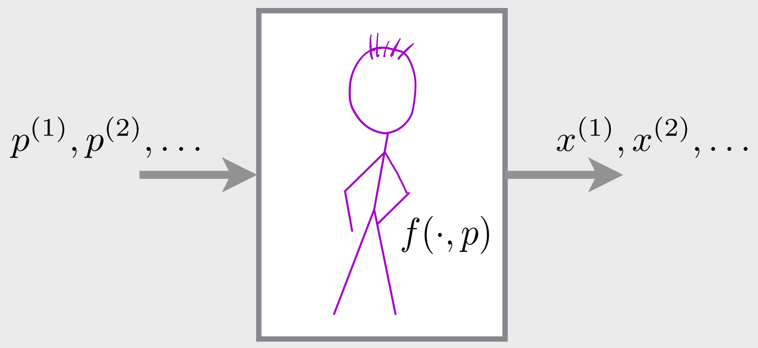

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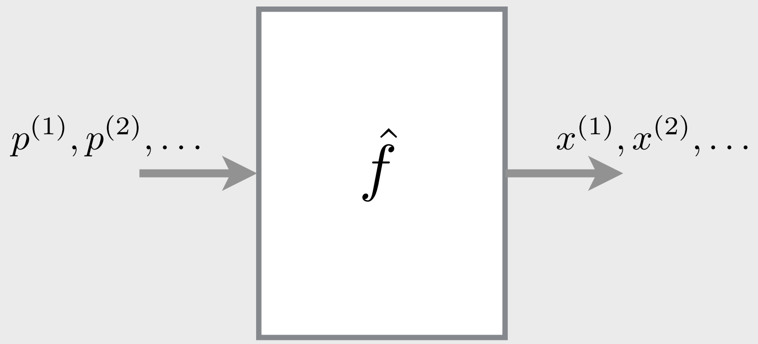
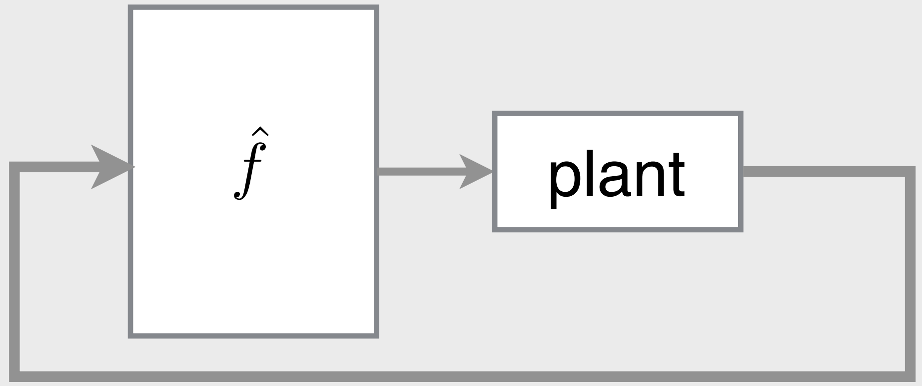


3. Learned objective (e.g. loss function) used to mimic the expert in an autonomous system



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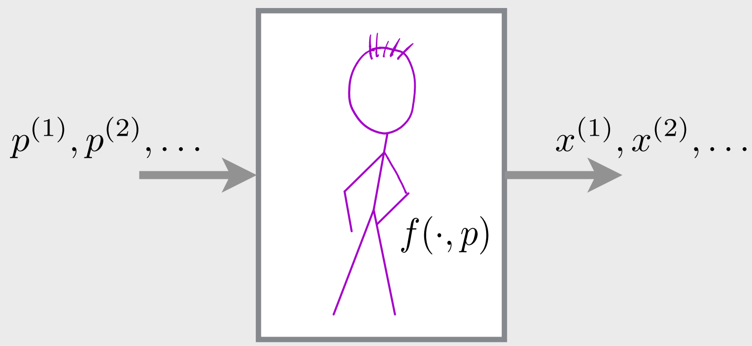

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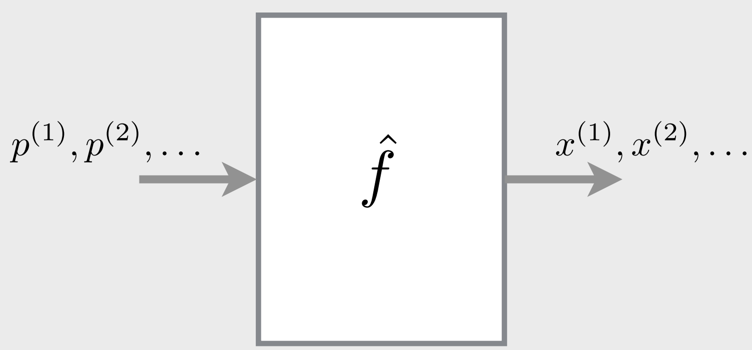

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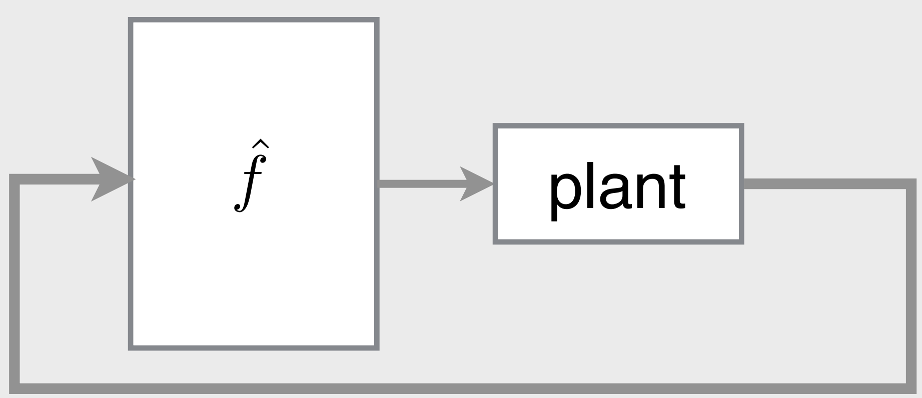
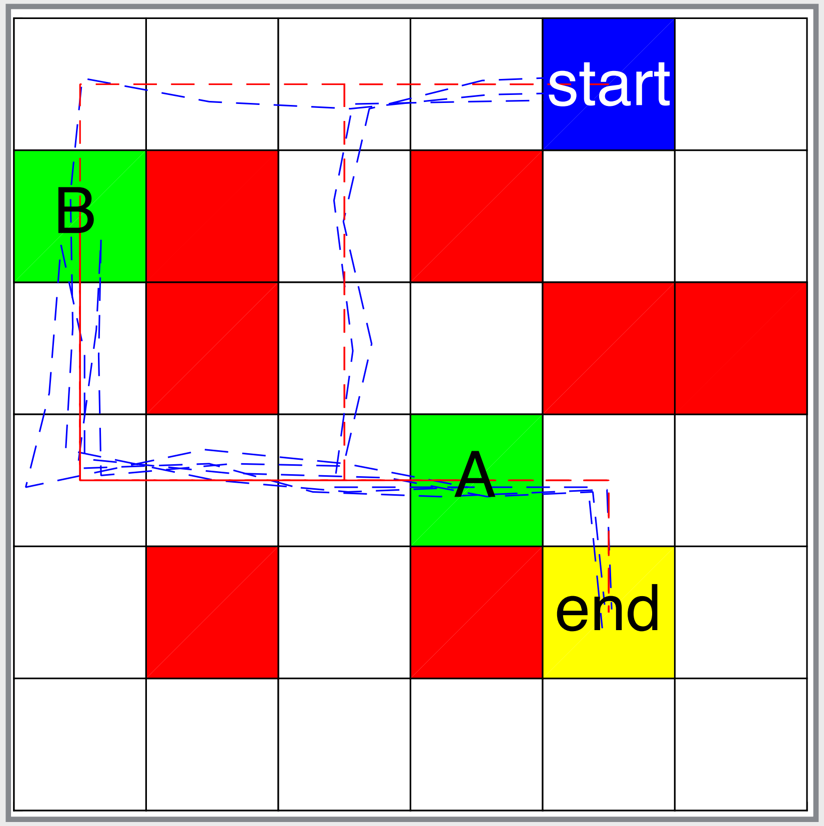


heli.stanford.edu

Learning fails when the expert is inconsistent

Expert demonstrations:



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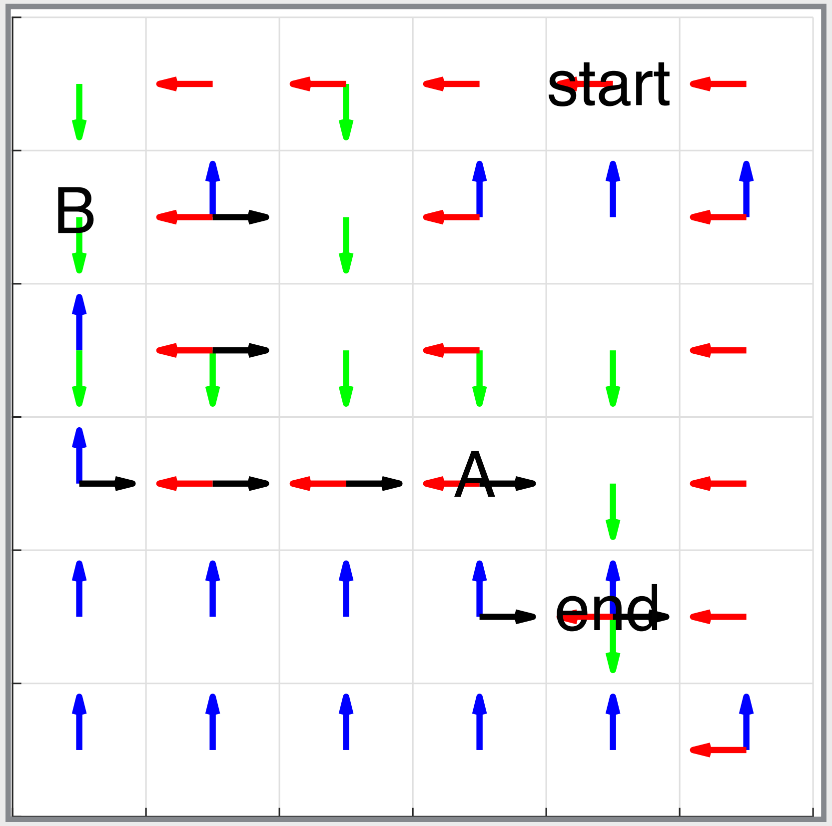

Task: get from start to end in the fewest steps, while visiting A and B in any order

- inverse optimal control applied to a grid world
- dynamics are modeled as a transition system
- learned an approximation to the optimal value function $V^\star(s)$

$$f(x, p) = \ell(s, s') + \hat{V}(s')$$

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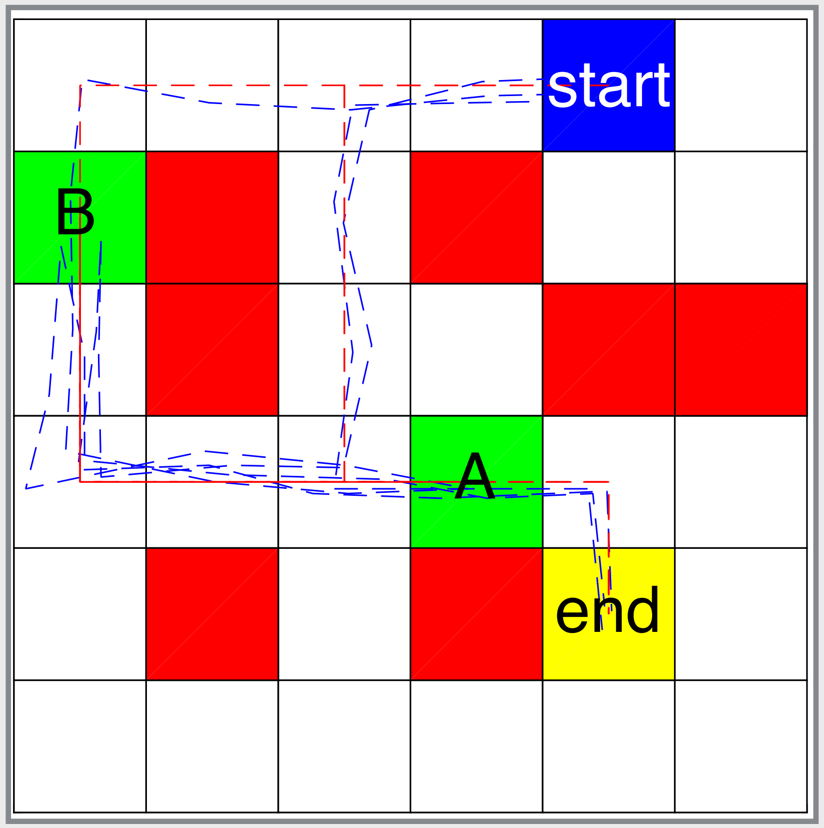

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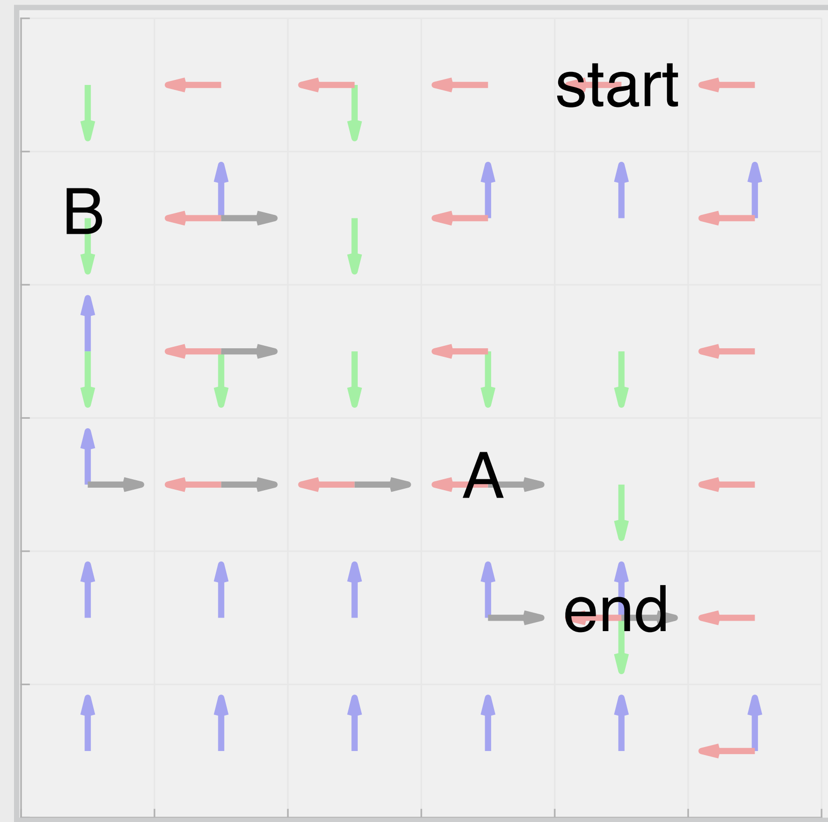

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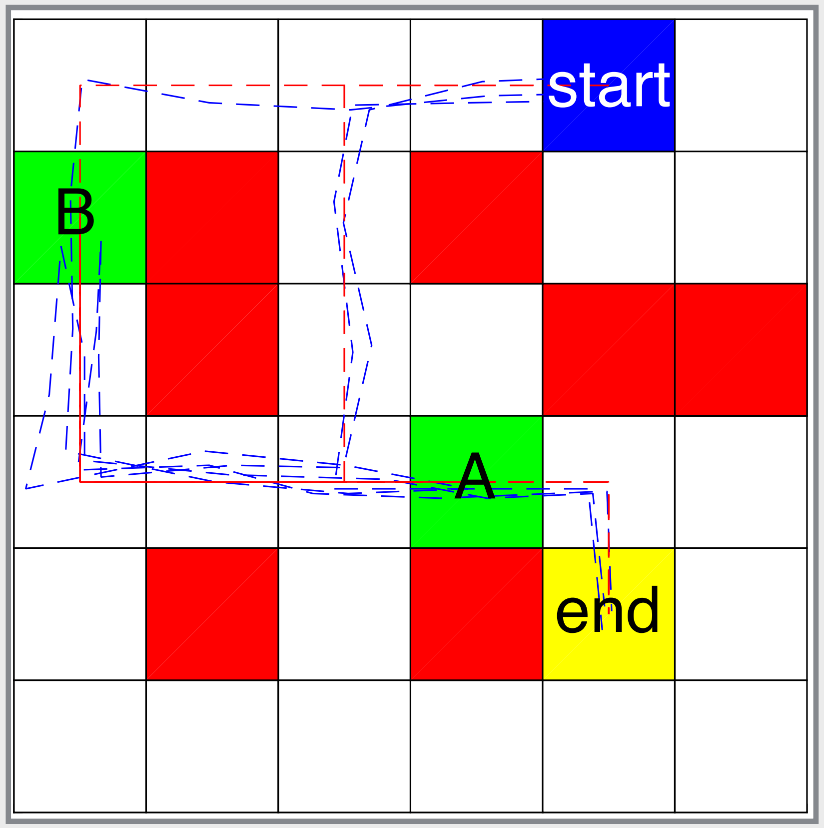

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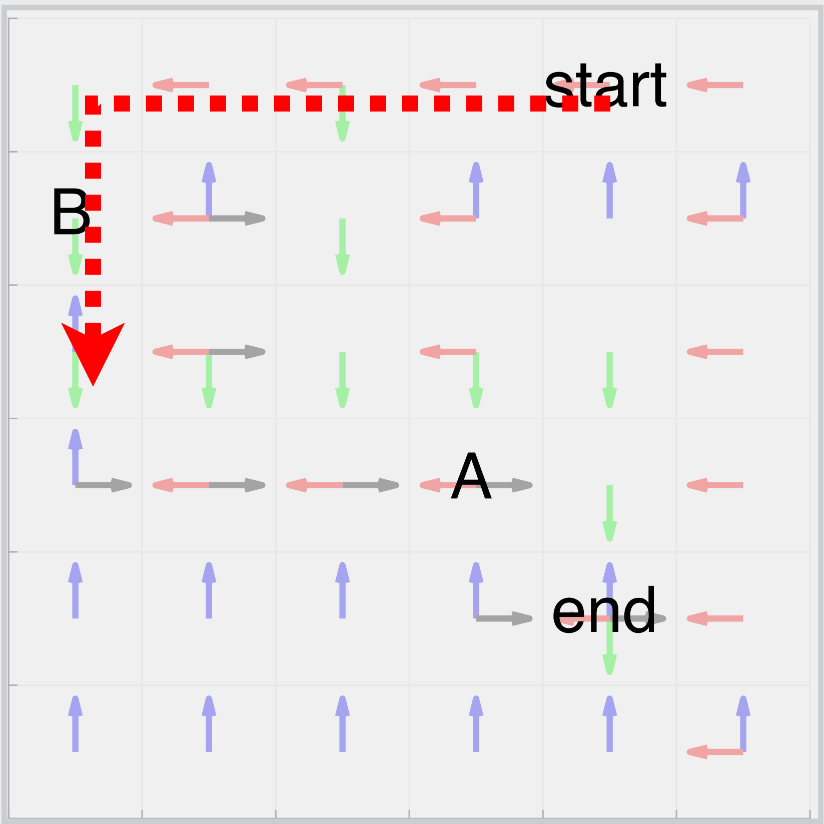

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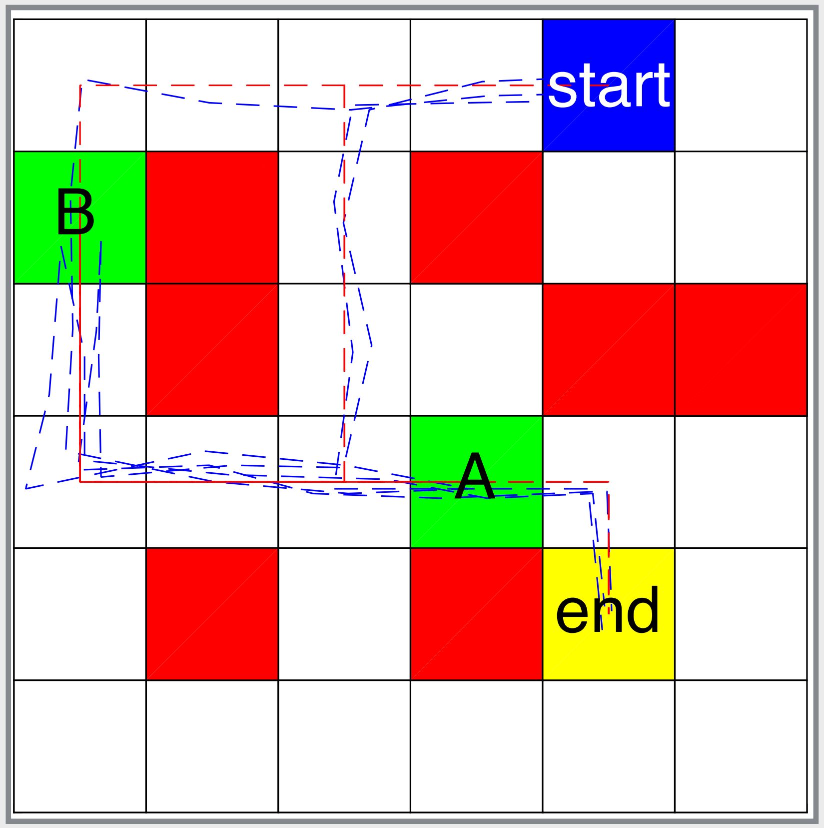

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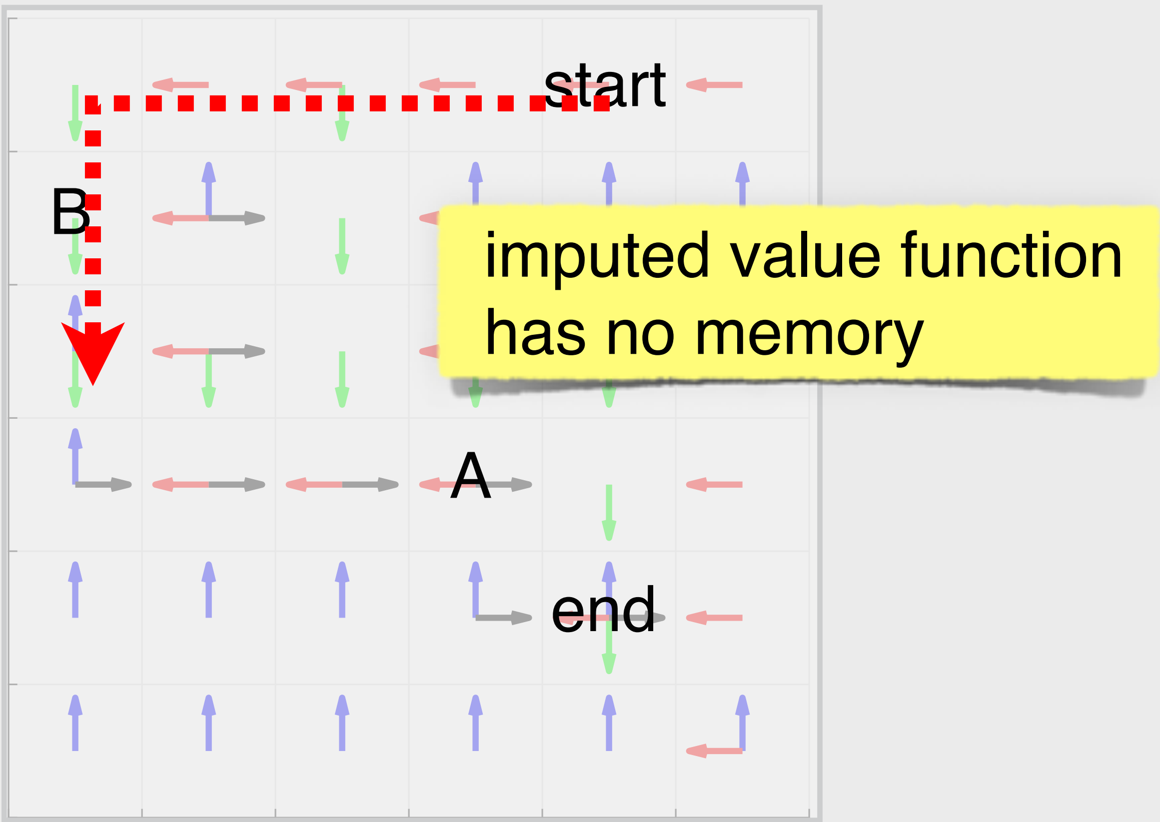

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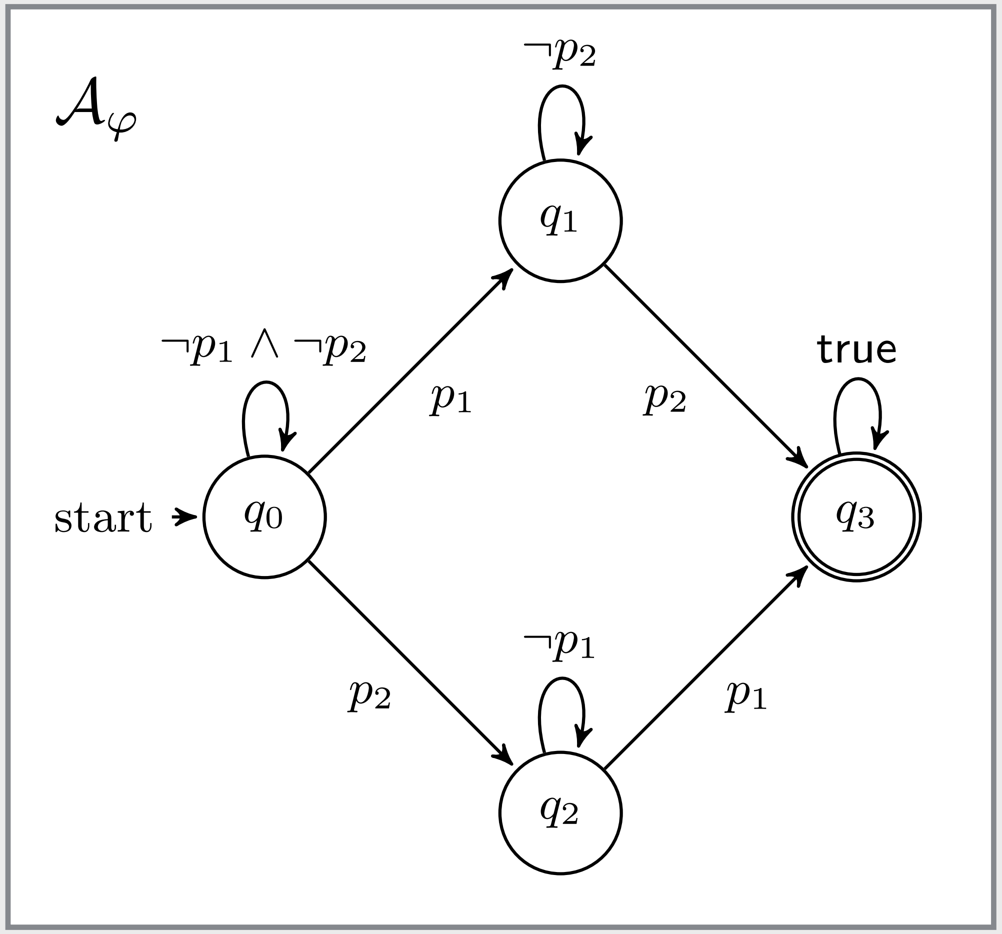
Side information

Task: get from start to end in the fewest steps, while visiting A and B in any order

Side information automaton



time	0	1	2	3	…	t	t+1
state	s_1	s_2	s_3	s_4	…	s_t	s_{t+1}
p_1	F	F	T	F		T	F
p_2	F	F	F	F		T	F

Side information: a specification automaton that every "optimal" trajectory must satisfy.

At each time step the atomic propositions p_1 and p_2 are evaluated

- p_1 = true iff. the state is A
- p_2 = true iff. the state is B

In the inverse problem, the side information becomes a **hidden state** with (known) evolution

Data structures

Memoryless policy:

$$\mu_t^\star(s) \in \operatorname*{argmin}_{\{\alpha \mid s \xrightarrow{\alpha} s'\}} \left\{ \ell(s,\alpha,s') + V_{t+1}^\star(s') \right\}$$

Mode-varying policy:

$$\mu^\star(s,q) \in \operatorname*{argmin}_{\{\alpha \mid s \xrightarrow{\alpha} s', q' = \delta(q, L(s))\}} \{ \ell(s,\alpha,s') + V^\star(s',q') \}$$

Expert data:

$$\mathcal{D} = \left\{ \Big((\alpha^{(k)}, {s'}^{(k)}, {q'}^{(k)}),\ (s^{(k)}, q^{(k)}) \Big) \mid k = 1, \ldots, N \right\}$$

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optimizing process

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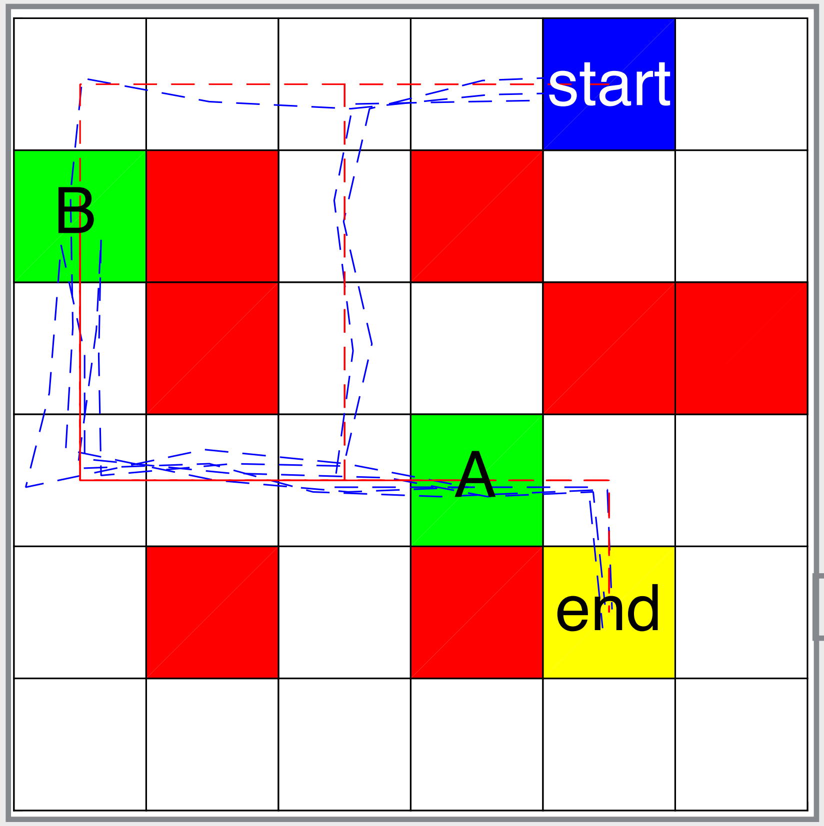
$x^{(k)}$ $p^{(k)}$

compare to memoryless case:

$$\mathcal{D} = \left\{ \left((\alpha^{(k)}, s'^{(k)}),\ s^{(k)} \right) \mid k = 1, \ldots, N \right\}$$

Side information-aware policy has memory

Expert demonstrations:



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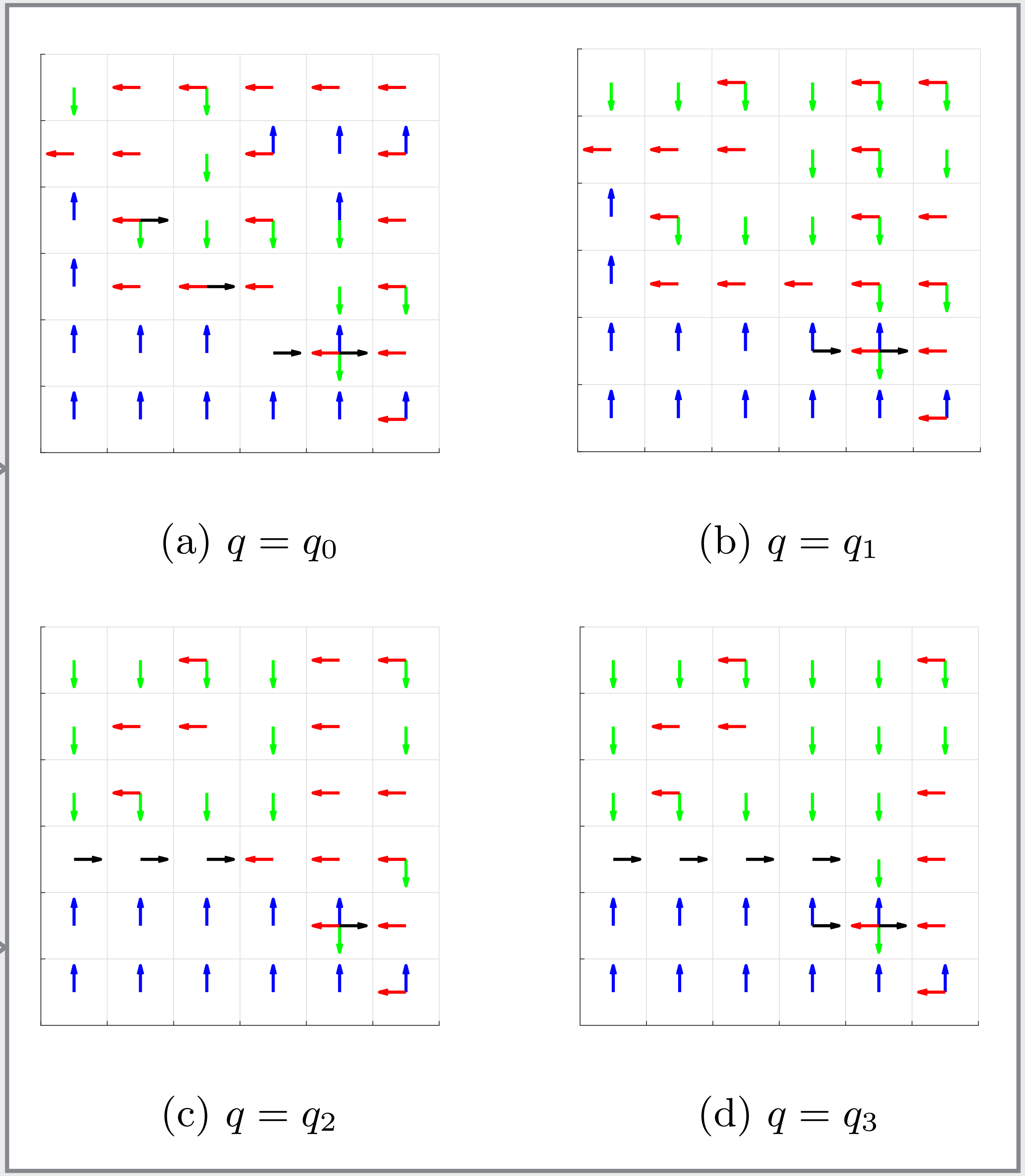

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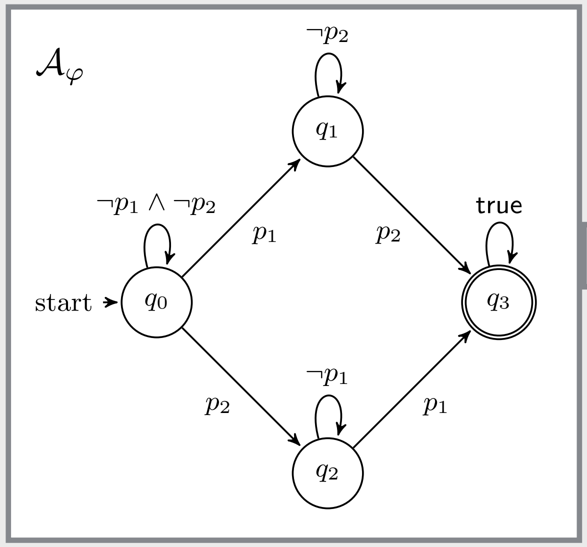

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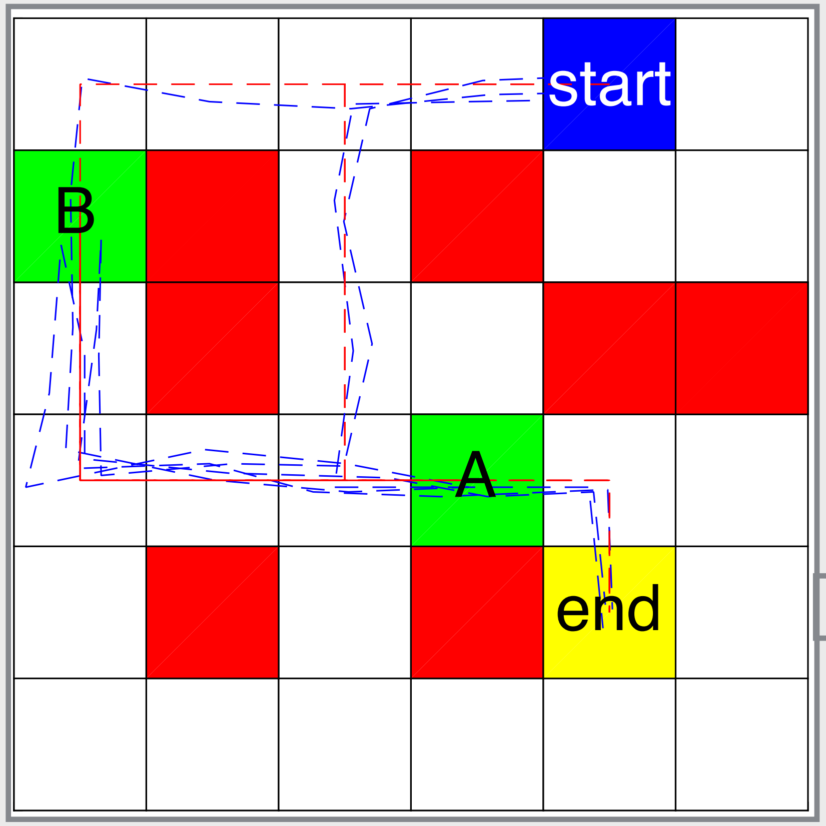

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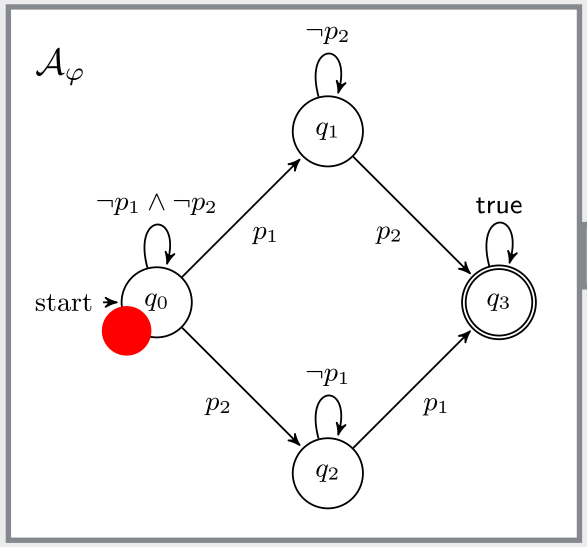

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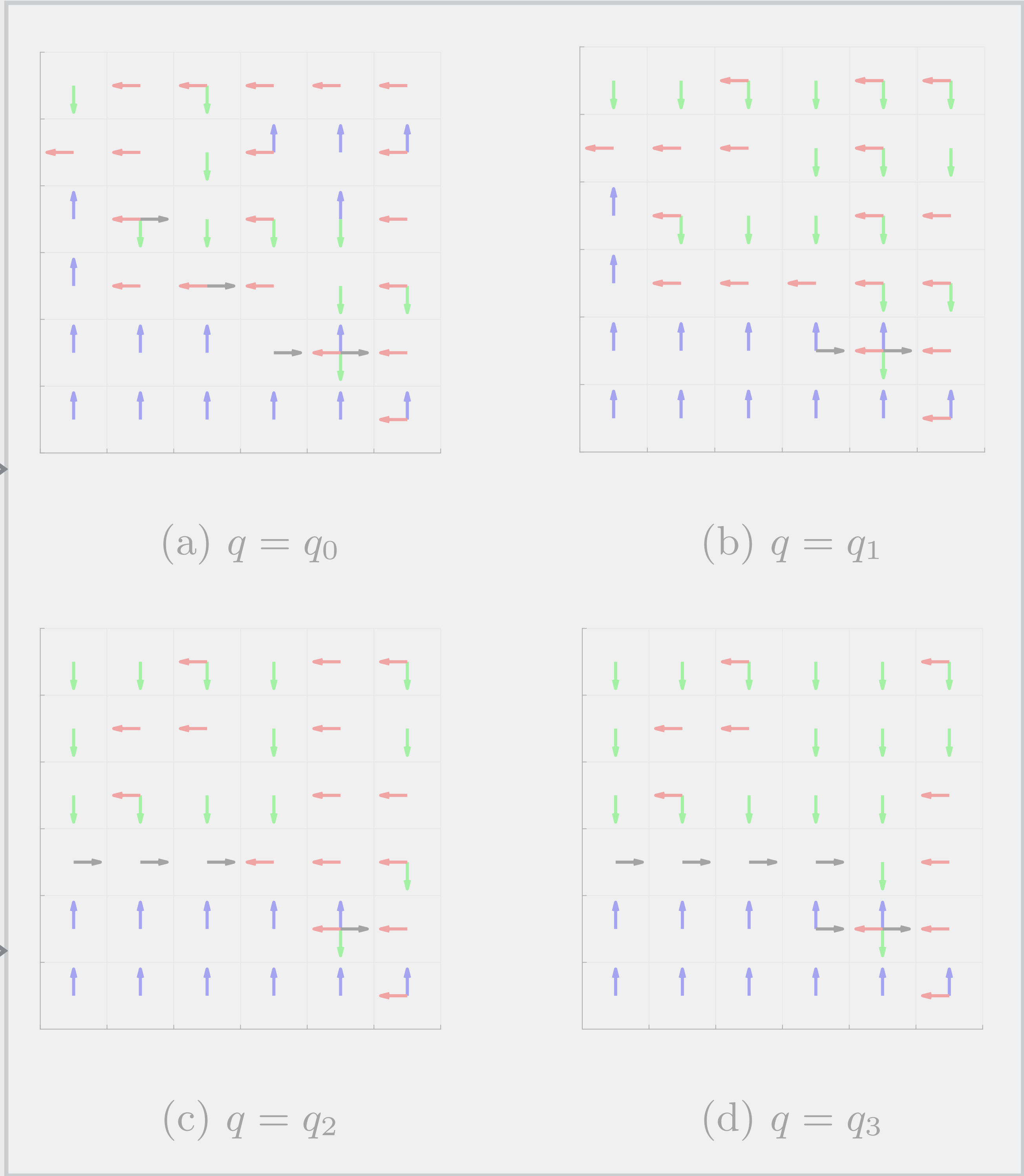

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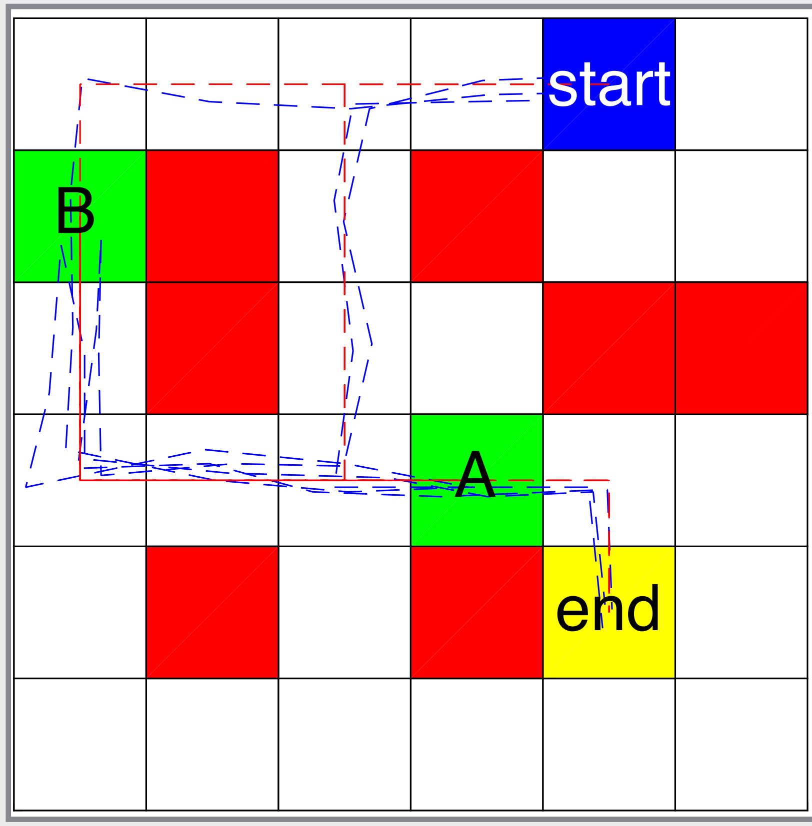

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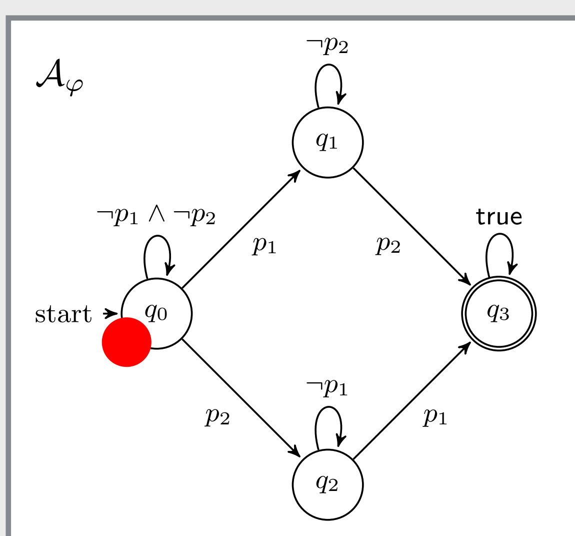

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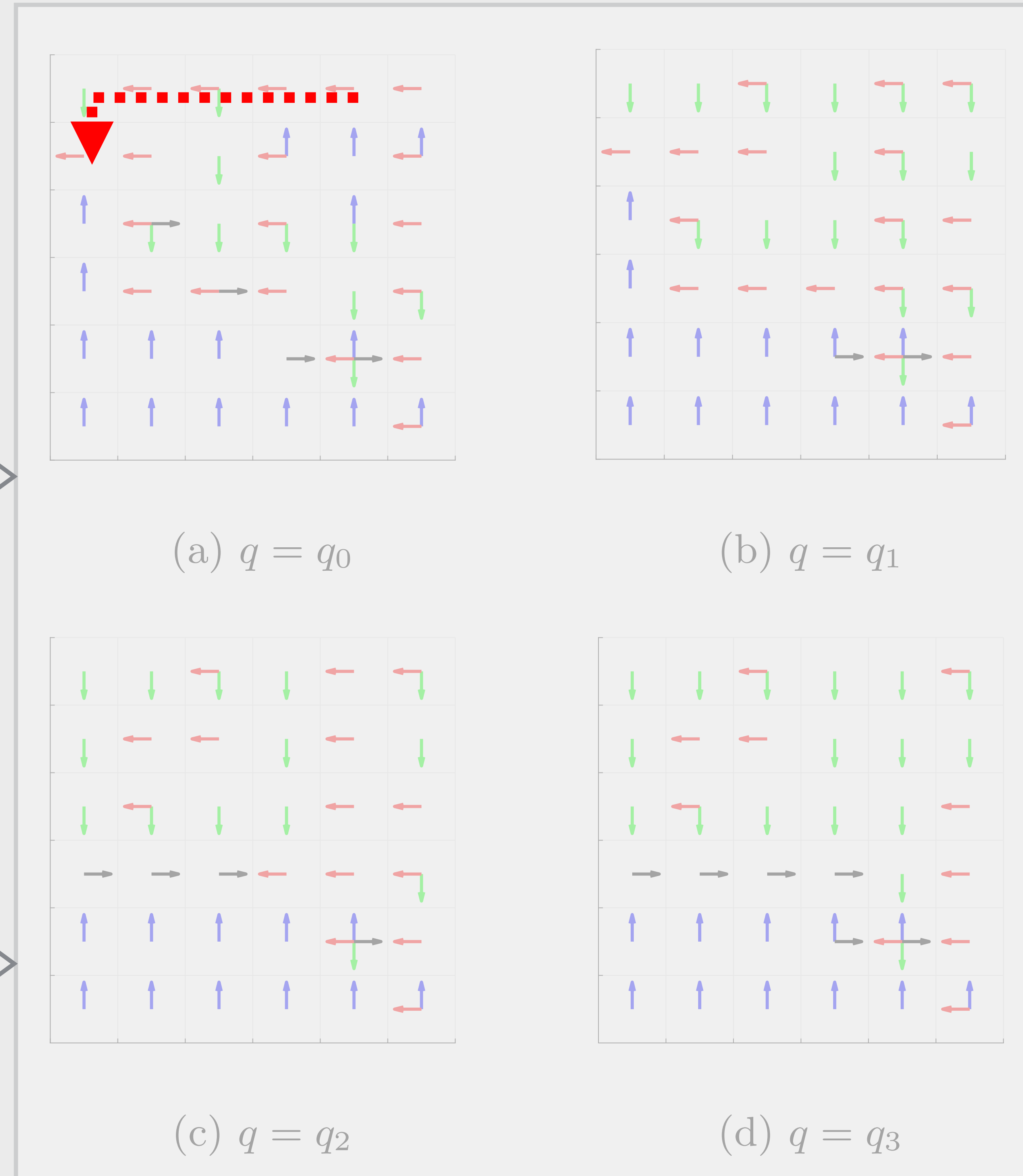

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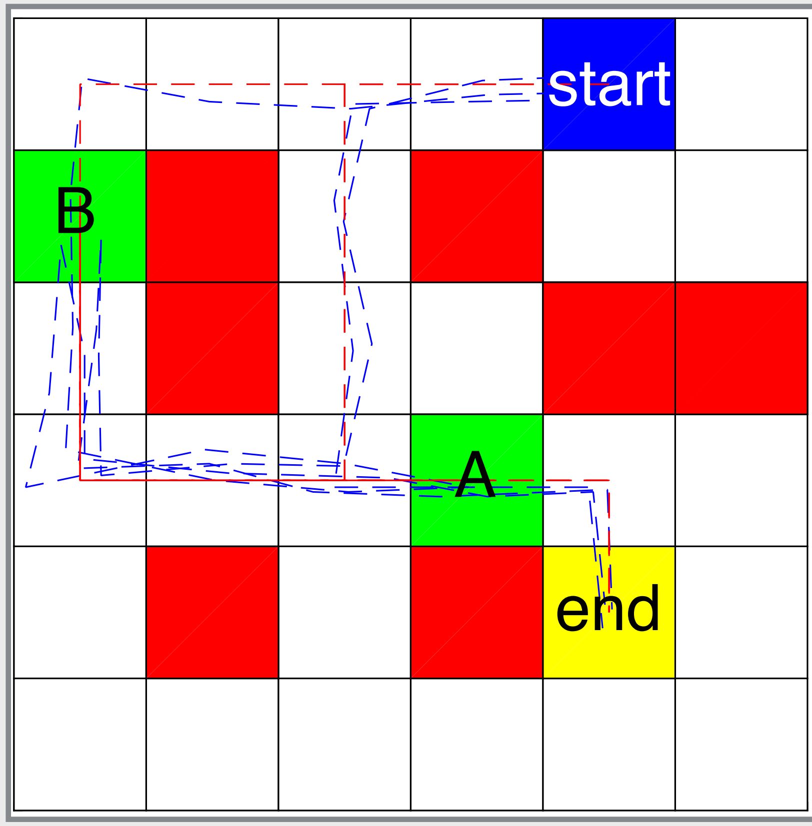

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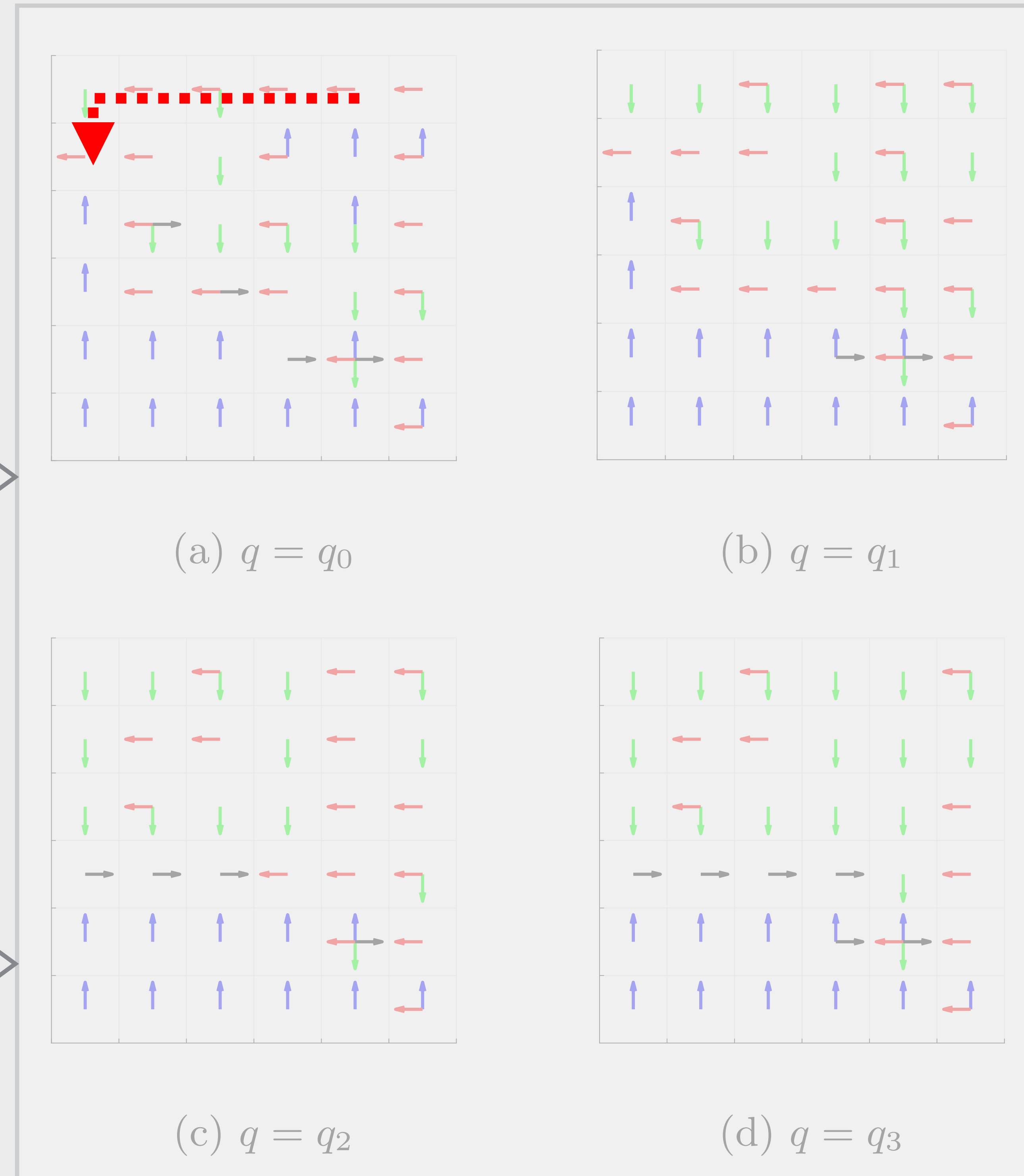

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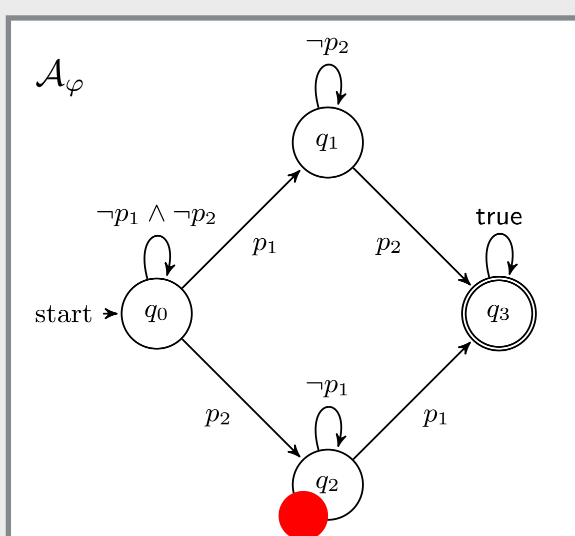

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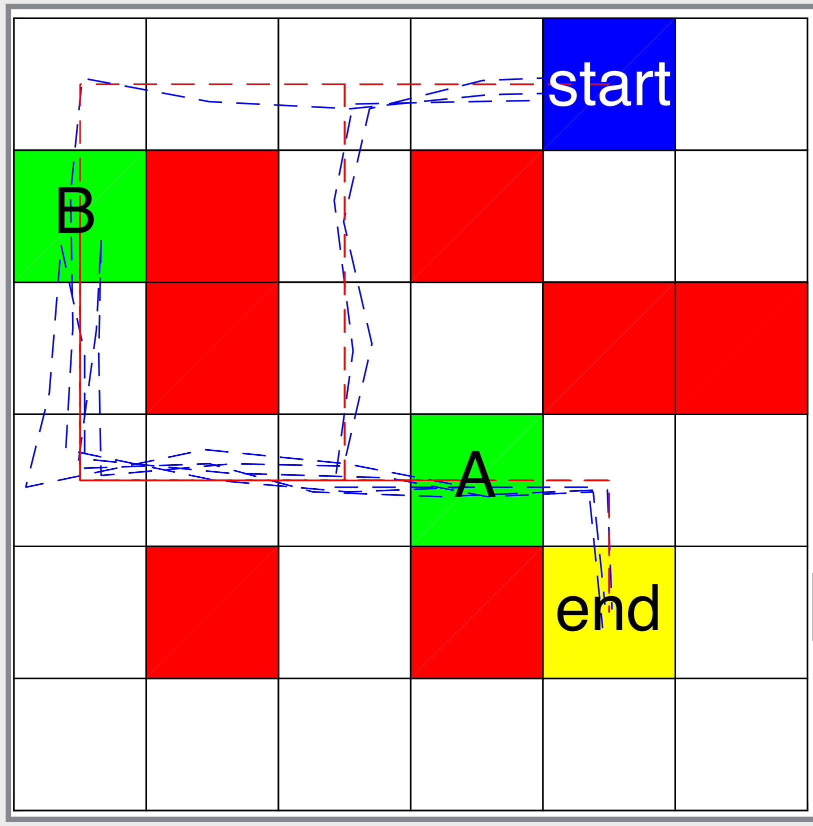

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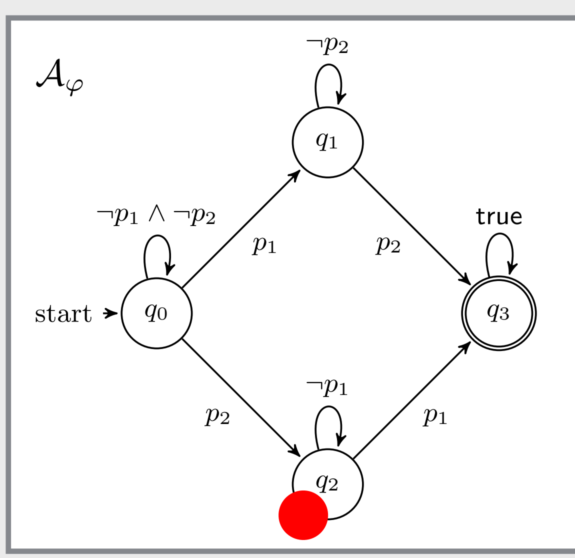

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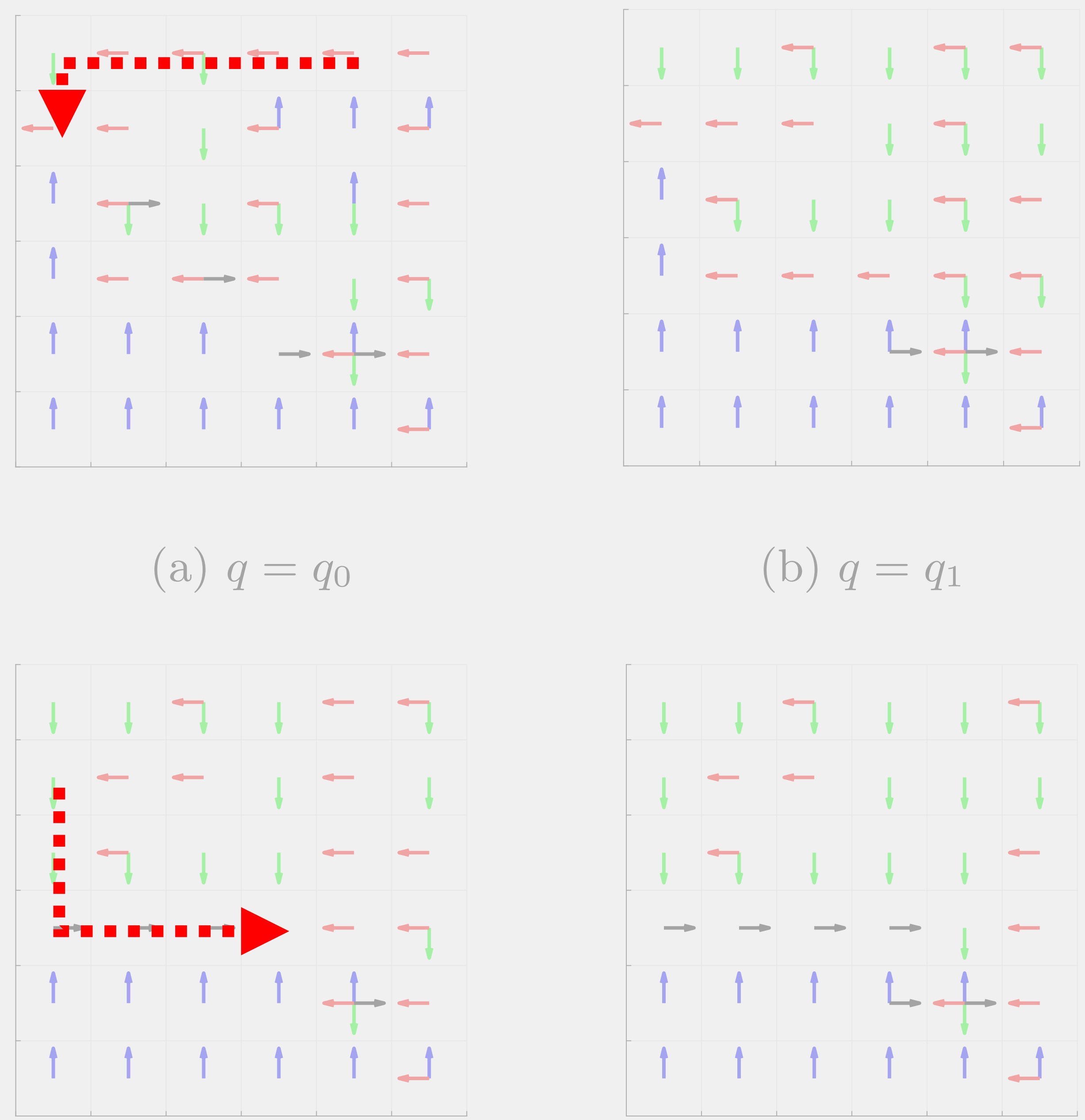

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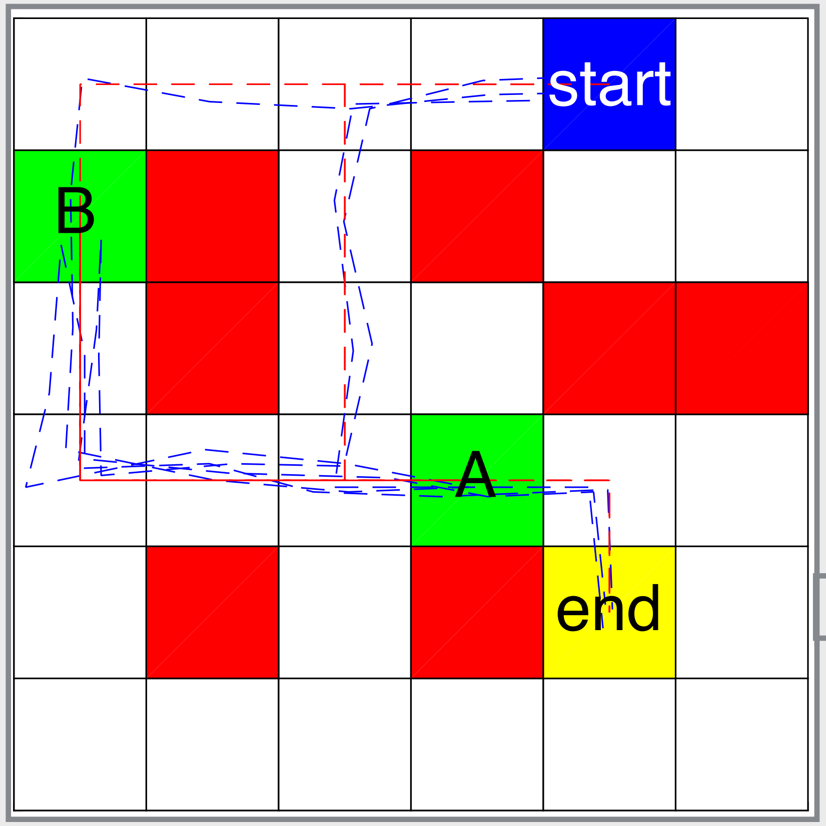

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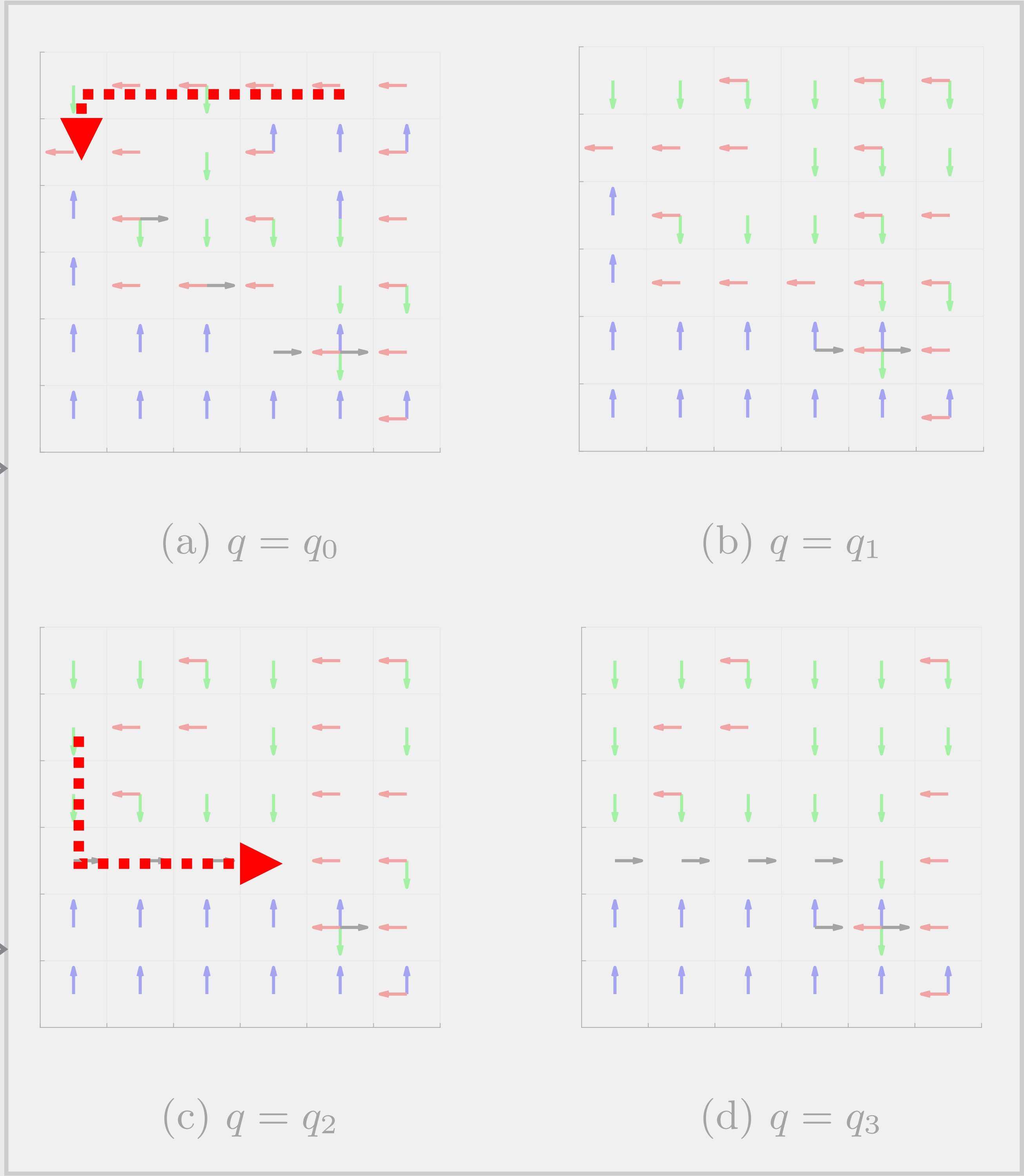

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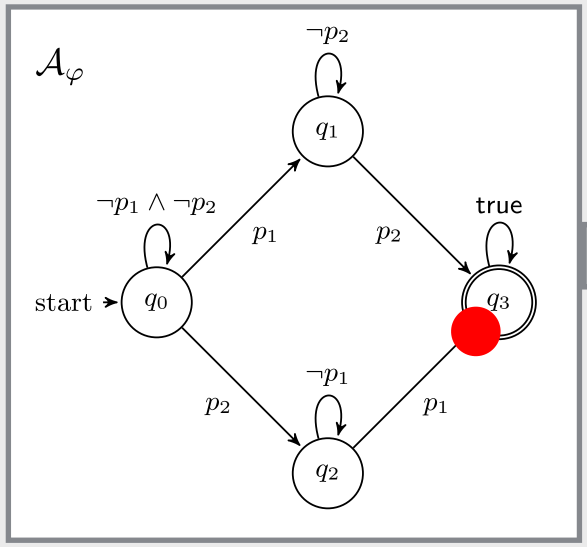

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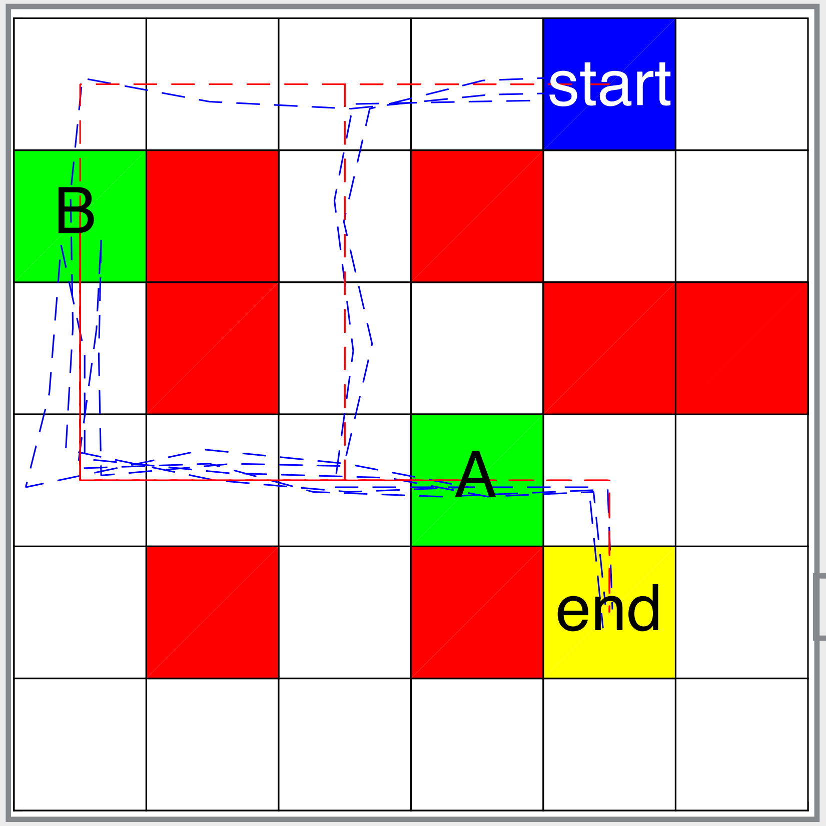
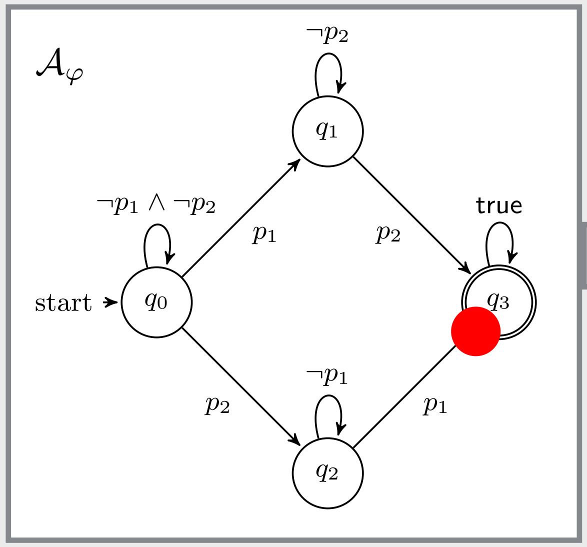
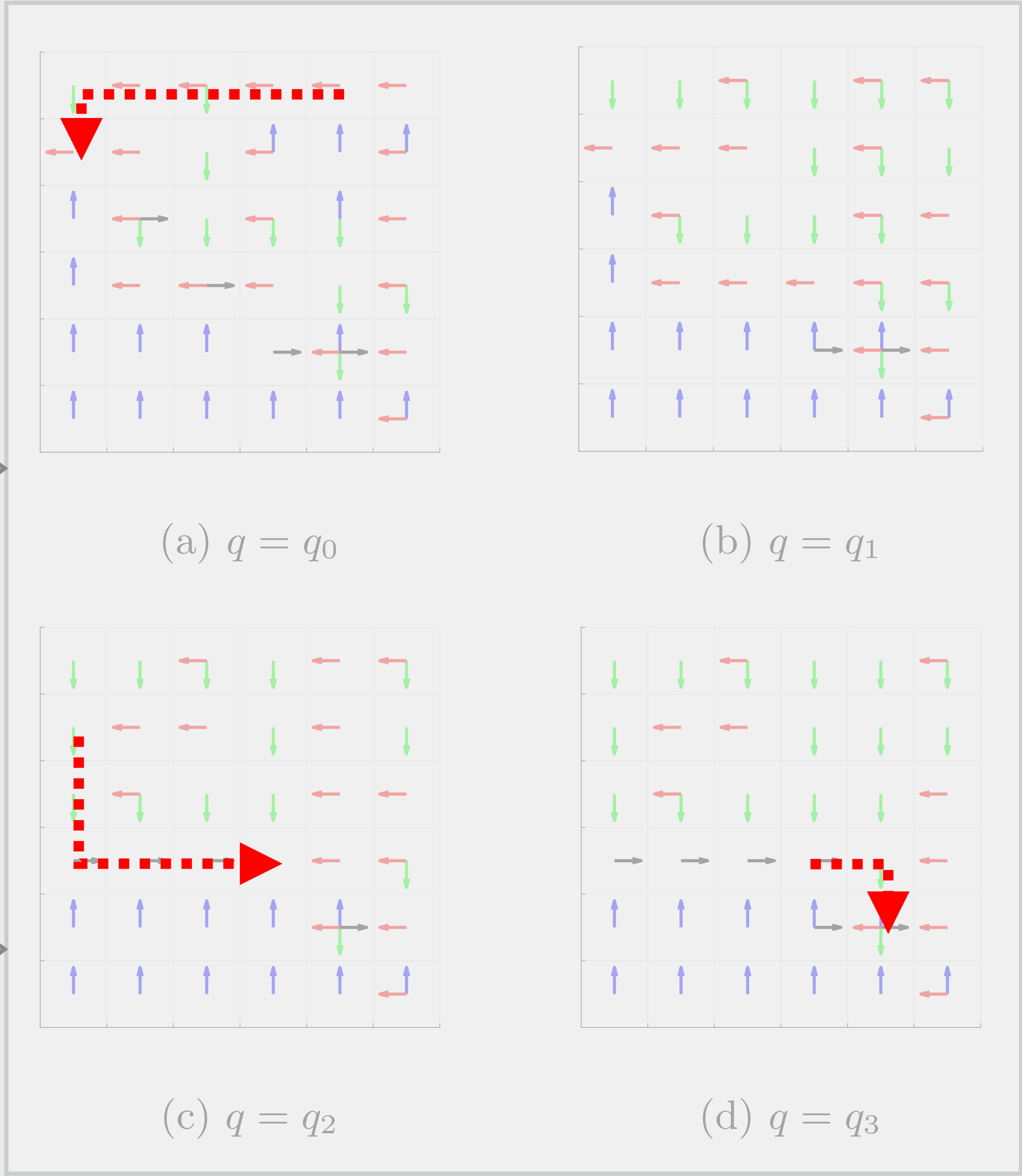


Side information:



Learned policy:



Summary

direct extensions

- Extend to broader dynamics classes—hybrid, nonlinear…
- Expand the family of specifications and languages
- Investigate the role of stochastic policies and partially specified side information
- Demonstrate scalability

new opportunities

- Open up a broad set of new problems to ideas from control and optimization

CDC 2016

Automata Theory Meets Approximate Dynamic Programming: Optimal Control with Temporal Logic Constraints

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Learning from Demonstrations with High-Level Side Information

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https://github.com/u-t-autonomous/sydar

SYDAR: Synthesis Done Approximately Right

Installation and usage

```
$ pip install sydar
$ sydar-matlab [input_file.miu] -o output.m
```

Acknowledgments

- Jie Fu (WPI)
- Min Wen (UPenn)
- Ufuk Topcu (UTexas)
- Richard Murray (Caltech)

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